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Some Observations Regarding Poisson's Ratios for Anisotropic Soils

Abstract. *The definition and determination of elastic moduli for soils are generally well-established. The same is not, however, necessarily true for Poisson's ratios. This paper thus reviews some key issues associated with determining Poisson's ratio values for soils. Although the discussion includes isotropic elastic material idealizations for completeness, the emphasis is placed on transversely isotropic elastic idealizations.*
Keywords. *Poisson's ratio, elasticity, isotropy, anisotropy, orthotropic, transversely isotropic.*

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1. Introduction. The definition and selection of Young's, shear and bulk modulus values for soils is generally well-established. The same cannot be said for Poisson's ratio. Indeed, the importance of this elastic parameter has largely been unappreciated [16]. Although the use of approximate or "typical" values of Poisson's ratio in soil mechanics applications may not necessarily pose significant difficulties, the value of Poisson's ratio undeniably affects the magnitude of elastic deformations of soil masses subjected to static and dynamic loading. This paper reviews some key issues associated with Poisson's ratios for soils. Although some of these issues have been discussed in previous papers and books, the focus has traditionally been on isotropic elastic material idealizations. While important findings related to isotropic idealizations are presented in this paper for completeness, the discussion emphasizes the evolving topic of transversely isotropic elastic material idealizations.

2. Basic Definition and a Historical Note. Poisson's ratio is defined as the negative of the ratio of transverse strain to the axial strain in an elastic material subjected to a uniaxial strain. In the mechanics of deformable bodies, the tendency of a material to expand or contract in a direction orthogonal to a loading direction is commonly referred to as the "Poisson's effect" [16].

Poisson's ratio is named after the French mathematician and physicist Simeon Denis Poisson (1787-1840), who first described this elastic constant in 1829 [57]. Gercek [16] and Greaves et al. [20] give historical details pertaining to the definition of Poisson's ratio. Biographical information for S. D. Poisson can be found in books by Timoshenko [64] and Todhunter and Pearson [69].

3. Elastic Constitutive Relations. For a general homogeneous, anisotropic linear elastic (Hookian) material, in the absence of initial strains and stresses, the constitutive relations, in "direct" vector-matrix form, are given by

$$\delta\varepsilon^{e} = A\ \delta\sigma' \qquad (1)$$

where A is a symmetric ($N_{rowb}*N_{rowb}$) matrix of compliance coefficients characterizing the material, and are ($N_{rowb}*1$) vectors of infinitesimal elastic strain and effective stress increments, respectively, and N_{rowb} is the number of stress and strain components (for three-dimensional analyses, $N_{rowb}=6$; for torsionless axisymmetry, $N_{rowb}=4$; for plane strain analyses, $N_{rowb}=3$). For three-dimensional analyses,

$$\delta\varepsilon^e = \{\delta\varepsilon_{11}^e \delta\varepsilon_{22}^e \delta\varepsilon_{33}^e \delta\gamma_{12}^e \delta\gamma_{13}^e \delta\gamma_{23}^e\}^T$$

$$\delta\sigma' = \left\{\delta\sigma_{11}^{'} \delta\sigma_{22}^{'} \delta\sigma_{33}^{'} \delta\sigma_{12}^{'} \delta\sigma_{13}^{'} \delta\sigma_{23}^{'}\right\}^T$$

where $\gamma_{12}^e, \gamma_{13}^e$, and γ_{23}^e are engineering shear strains, and the superscript T denotes the operation of vector transposition.

Written in "inverse" vector-matrix form, the constitutive relations are given by generalized Hooke's law; viz.,

$$\boldsymbol{\delta\sigma' = D\delta\varepsilon^e} \qquad (2)$$

where **D**, which is the inverse of **A**, represents the symmetric *(N_{rowb} ∗ N_{rowb})* matrix of elastic moduli.

Due to symmetry, in their most general form, **A** and **D** contain 21 independent coeffcients that characterize the elastic material. Fortunately, however, most of the important engineering materials possess some internal structure that exhibits certain symmetries that reduce the number of required coeffcients. Books by Love [49] and Lekhnitskii [44] give additional details pertaining to this subject.

4. Consideration of Elastic Isotropy. Most natural soils exhibit some degree of anisotropy due to their manner of deposition, particle shape, and stress history [8; 3; 18; 17]. For example, sedimentary soils, which are typically deposited under gravity, possess different properties in the direction of deposition as opposed to the planes normal to this direction.

Traditionally, however, the elastic response of soils has been assumed to be isotropic. This was primarily done for two reasons. First, was a desire not to overly complicate analytical formulations. Second, was the lack of suitable experimental equipment to measure the elastic constants necessary to characterize the anisotropic elastic response of soils. The consideration of elastic material idealizations thus begins with the special case isotropic elasticity.

5. Isotropic Elastic Idealizations. Limited experimental results on several different sands [60] and on kaolin clay [37] indicate isotropic behavior upon unloading, even when the strains during loading indicated anisotropic behavior. Results for sensitive clays also showed nearly isotropic elastic behavior, though the associated plastic stress-strain relations were, however, anisotropic [71]. Citing the above results for sands and clays, Lade and Nelson [43] concluded that although microscopic elastic behavior of geomaterials is randomly anisotropic and non-homogeneous, such materials can be considered as macroscopically homogeneous and isotropic.

To represent the constitutive relations in vector/matrix form, the compliance matrix appearing in Equation (1) is written in terms of (3∗3) sub-matrices, giving

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A_{11}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A_{22}} \end{bmatrix} \qquad (3)$$

Expressing an isotropic elastic material in terms of the "drained" Young's modulus E' and Poisson's ratio ν',

$$\boldsymbol{A_{11}} = \frac{1}{E'}\begin{bmatrix} 1 & -\nu' & -\nu' \\ -\nu' & 1 & -\nu' \\ -\nu' & -\nu' & 1 \end{bmatrix} \text{ and } \boldsymbol{A_{22}} = \frac{1}{E'}\begin{bmatrix} 2(1+\nu') & 0 & 0 \\ 0 & 2(1+\nu') & 0 \\ 0 & 0 & 2(1+\nu') \end{bmatrix} \qquad (4)$$

where it is noted that $A_{44} = A_{55} = A_{66} = 2(A_{11} - A_{12})$.

If the isotropic elastic material is instead represented by the drained bulk modulus K' and the shear modulus G, then A_{11} and A_{22} become

$$A_{11} = \begin{bmatrix} \left(\frac{1}{9K'}+\frac{1}{3G}\right) & \left(\frac{1}{9K'}-\frac{1}{6G}\right) & \left(\frac{1}{9K'}-\frac{1}{6G}\right) \\ \left(\frac{1}{9K'}-\frac{1}{6G}\right) & \left(\frac{1}{9K'}+\frac{1}{3G}\right) & \left(\frac{1}{9K'}-\frac{1}{6G}\right) \\ \left(\frac{1}{9K'}-\frac{1}{6G}\right) & \left(\frac{1}{9K'}-\frac{1}{6G}\right) & \left(\frac{1}{9K'}+\frac{1}{3G}\right) \end{bmatrix} \text{ and } A_{22} = \frac{1}{G}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

6. Orthotropic Elastic Idealizations. Over the last 35 or so years, substantial progress has been made in the development of experimental techniques that facilitate the measurement of the aforementioned elastic constants. Such measurements confirm that soils indeed exhibit elastic response, albeit at low strain levels, and that this response is typically anisotropic. Consequently, anisotropic elastic material idealizations for soils have become significantly more tractable.

The description of anisotropic elastic materials is complicated by the fact that elastic moduli $\left(E_i^{'}\right)$ are associated with a single direction of stretch, and thus require only a single subscript. The classification of Poisson's ratios $\left(\nu_{ij}^{'}\right)$ and the shear moduli $\left(G_{ij}\right)$ depends upon pairs of orthogonal directions. Consequently, these elastic constants require a pair of subscripts; these subscripts do not, however, follow the rules associated with indicial notation. The consideration of anisotropic elastic material idealizations begins with the special case of orthotropic elasticity.

Consider a material through each point of which pass three mutually perpendicular planes of elastic symmetry. If similar planes are parallel at all points in the material, then taking the $\left(x_1, x_2, x_3\right) \equiv \left(x, y, z\right)$ coordinate axes normal to these planes (i.e., along the principal directions) it follows that there should be no interaction between the various shear components or between the shear and normal components. Consequently, the compliance matrix has the following entries [44]:

$$A = \begin{bmatrix} 1/E_1^{'} & -\nu_{21}^{'}/E_2^{'} & -\nu_{31}^{'}/E_3^{'} & 0 & 0 & 0 \\ -\nu_{12}^{'}/E_1^{'} & 1/E_2^{'} & -\nu_{32}^{'}/E_3^{'} & 0 & 0 & 0 \\ -\nu_{13}^{'}/E_1^{'} & -\nu_{23}^{'}/E_2^{'} & 1/E_3^{'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \quad (6)$$

The material constants appearing in Equation (6) are defined as follows: $E_1^{'}, E_2^{'}$, and $E_3^{'}$ are elastic moduli associated with tension or compression in the material coordinate direction x_1, x_2, and x_3, respectively. These moduli are obtained under drained conditions; they are thus defined in terms of effective stress. The G_{ij} is the elastic shear modulus that relates the shear stress σ_{ij} to the shear strain ν_{ij}, where no summation on repeated indices is implied. Finally, $\nu_{ij}^{'}$ is the Poisson's ratio that is equal to the ratio of the lateral contraction in the x_j material coordinate direction resulting from a uniaxial extension in the x_i coordinate direction [44]. The Poisson's ratios are also obtained under drained conditions; thus, similar to $E_1^{'}, E_2^{'}$, and $E_3^{'}$, they are likewise defined in terms of *effective stress.*

Symmetry of **A** implies that $\nu_{21}^{'}/E_2^{'} = \nu_{12}^{'}/E_1^{'}$, $\nu_{21}^{'}/E_2^{'} = \nu_{12}^{'}/E_1^{'}$, and $\nu_{32}^{'}/E_3^{'} = \nu_{23}^{'}/E_2^{'}$. Thus, only nine of the twelve elastic constants entering Equation (6) are independent; i.e., $E_1^{'}, E_2^{'}, E_3^{'}, \nu_{12}^{'}, \nu_{13}^{'}, \nu_{23}^{'}, G_{12}, G_{13}, G_{23}$.

Although some experimental findings [35] suggest that natural soils are elastically orthotropic, the difficulty associated with determining values for the nine elastic constants precludes the adoption of such an idealization. Instead, transversely isotropic elasticity is commonly assumed, thus reducing the number of elastic parameters to *five*.

7. Transversely Isotropic Idealizations. Due to the manner in which natural soils are deposited, it is logical to expect them to exhibit approximately transversely isotropic (or "cross-anisotropic") response. While this realization is not new [3; 56; 2; 19; 61], the lack of suitable experimental apparatus to accurately measure the five elastic constants associated with transverse isotropy has, in the past, precluded the use of such idealizations. More recently [29; 47; 40; 1; 53], substantial progress has been made in experimental techniques that facilitate the measurement of the aforementioned elastic constants.

Through all points of a transversely isotropic material there pass parallel planes of elastic symmetry in which all directions are elastically equivalent (i.e., planes of isotropy). Thus at each point there exists one principal direction and an infinite number of principal directions in a plane normal to the first direction [44]. Assume that the local material axes $(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \equiv (\tilde{x}, \tilde{y}, \tilde{z})$ coincide with the global x, y and z coordinate axes (Figure 1). Furthermore, assume that the global $x_1 \equiv x$ - axis is taken normal to the planes of isotropy, with the global y and z axes directed arbitrarily in such planes.

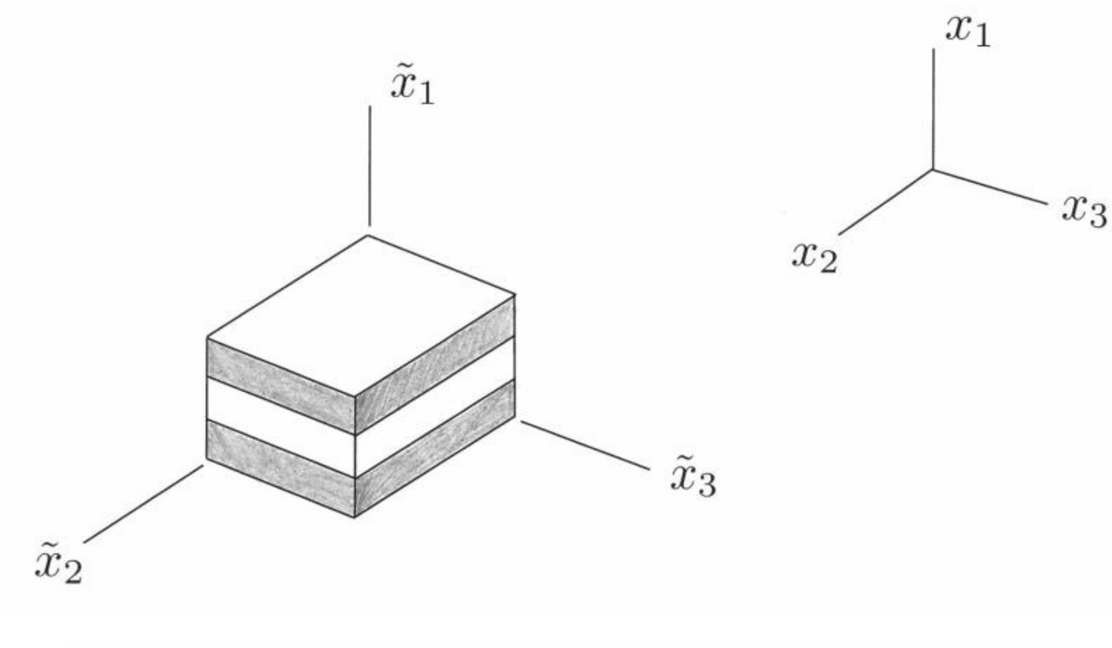

Figure 1: Schematic illustration of an element of transversely isotropic material.

In light of this definition of a transversely isotropic material, and in relation to the orthotropic elastic constants given in Equation (6), the following elastic constants are defined: $E_1^{'} \equiv E_n^{'}$ where $E_n^{'}$ is the elastic modulus for compression or tension in a direction normal to the plane of isotropy, and $E_2^{'} = E_3^{'} \equiv E_t^{'}$, where $E_t^{'}$ is the elastic modulus for compression or tension in the plane of isotropy (i.e., in a direction tangential to the plane of isotropy). Since the y - z plane is a plane of isotropy, $\nu_{21}^{'} = \nu_{31}^{'} \equiv \nu_{tn}^{'}$, where $\nu_{tn}^{'}$ is the Poisson's ratio characterizing the lateral contraction normal to the plane of isotropy when tension is applied in the plane. The modulus $G_{12} = G_{13} \equiv G_{nt}$ is associated with shearing involving $\delta\gamma_{12}^{e}$ and $\delta\gamma_{13}^{e}$. Finally, $G_{23} \equiv G_{tt}$ characterizes shearing in the plane of isotropy. It is given by $1/G_{tt} = 2(1+\nu_{tt}^{'})/E_t^{'}$, from which it is evident that G_{tt} is thus not an independent material constant.

From Equation (6), symmetry of **A** requires that $A_{23} = A_{32}$, giving $\nu_{32}^{'}/E_3^{'} = \nu_{23}^{'}/E_2^{'}$. Since $E_2^{'} = E_3^{'}$, it follows that $\nu_{32}^{'} = \nu_{23}^{'} \equiv \nu_{tt}^{'}$, where $\nu_{tt}^{'}$ is the Poisson's ratio characterizing transverse contraction in the plane of isotropy when tension is applied in the same plane.

Symmetry considerations also require that $A_{12} = A_{21}$ and $A_{13} = A_{31}$, giving $\nu_{21}^{'}/E_2^{'} = \nu_{12}^{'}/E_1^{'}$ and $\nu_{31}^{'}/E_3^{'} = \nu_{13}^{'}/E_1^{'}$. Since $\nu_{31}^{'} = \nu_{21}^{'}$, it follows that now $\nu_{13}^{'}/E_1^{'} = \nu_{12}^{'}/E_1^{'}$, thus giving $\nu_{12}^{'} = \nu_{13}^{'} \equiv \nu_{nt}^{'}$. Here ν_{nt} is the Poisson's ratio characterizing the lateral contraction in the plane of isotropy when tension is applied normal to the plane.

When the global x-axis is taken normal to the planes of isotropy, a transversely isotropic material is thus characterized by the values of five material constants, namely, $E_t^{'}, E_n^{'}, \nu_{nt}^{'}, \nu_{tt}^{'}, G_{nt}$. The compliance matrix given by Equation (6) thus becomes

$$\boldsymbol{A}=\begin{bmatrix} 1/E_n^{'} & -\nu_{tn}^{'}/E_t^{'} & -\nu_{tn}^{'}/E_t^{'} & 0 & 0 & 0 \\ -\nu_{nt}^{'}/E_n^{'} & 1/E_t^{'} & -\nu_{tt}^{'}/E_t^{'} & 0 & 0 & 0 \\ -\nu_{nt}^{'}/E_n^{'} & -\nu_{tt}^{'}/E_t^{'} & 1/E_t^{'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{nt} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{nt} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu_{tt}^{'})/E_t^{'} \end{bmatrix} \qquad (7)$$

where, from symmetry of **A**, $\nu_{tn}^{'}=\nu_{nt}^{'}\left(E_t^{'}/E_n^{'}\right)$.

Additional details pertaining to anisotropic elasticity for soils, including specific expressions for volumetric strains, response under undrained conditions, specialization for plane strain idealizations, and response under axisymmetric triaxial conditions are presented by Kaliakin [34].

8. Limiting Values for Elastic Material Parameters. For isotropic elastic material idealizations, the drained Young's modulus (E') is related to the shear modulus *(G)* and to the drained Poisson's ratio (ν') through the relation $E'=2G(1+\nu')$. No such relations are possible for anisotropic elastic materials, as the moduli and Poisson's ratios become dependent upon the directions of stretch, lateral strain, and the directions of shear.

The task of determining extreme values of the elastic moduli is somewhat simplified by the fact that such moduli depend only on a single direction of stretch (and thus require only a single subscript). Consequently, a detailed analysis of the extreme elastic moduli for cubic and transversely isotropic materials [9], and for materials with tetragonal symmetry [10] has been presented. Analytic expressions related to extrema of the elastic moduli have also been developed [5], as have been explicit expressions for the stress directions and the stationary values of Young's moduli for triclinic and monoclinic materials [66] and ones for orthotropic, tetragonal, trigonal, hexagonal and cubic materials [67].

Since they depend upon pairs of orthogonal directions (and thus require a pair of subscripts that do not follow the rules associated with indicial notation), the determination of limiting values for Poisson's ratio and the shear moduli is more complicated than for the Young's moduli. The following subsections thus discuss the extreme values of these elastic constants.

8.1 Orthotropic Material Idealizations. Although all nine material constants associated with an orthotropic elastic material idealization are independent, there are bounds on the values that these constants can assume. For an elastic material, thermodynamics requires that the strain energy must be positive. This will be achieved if the strain energy per unit volume $\left(\frac{1}{2}\dot{\sigma}^T A\sigma\right)$ is positive definite.

Lempriere [45] appears to have been the first to investigate the limiting values for the material constants associated with an orthotropic elastic material idealization. Considering various possible stress states, Lempriere [45] deduced that the diagonal entries in **A** must all be positive; viz.,

$$E_1^{'}>0, E_2^{'}>0, E_3^{'}>0, G_{12}>0, G_{13}>0, G_{23}>0 \qquad (8)$$

Noting that the elastic matrices **A** and **D** are symmetric and must be positive definite for the strain energy density to be positive, Ting [65] subsequently showed that, for the case of **A**, this condition will be satisfied provided that $A_{11}>0, A_{22}>0, A_{33}>0, A_{44}>0, A_{55}>0$ and $A_{66}>0$. This more general and rigorous treatment confirmed the earlier finding of Lempriere [45] given by Equation (8).

From the inverse form of the elastic constitutive relations given by Equation (2), Lempriere [45] argued that for uniaxial extensional strain states, the diagonal entries in **D** must all be positive. This leads to the following restriction on $\nu_{21}^{'}$:

$$\left(1-\nu_{12}^{'}\nu_{21}^{'}\right)=1-\left(\frac{E_1^{'}}{E_2^{'}}\nu_{21}^{'}\right)\nu_{21}^{'}>0 \;\Rightarrow\; \left(\nu_{21}^{'}\right)^2<\frac{E_1^{'}}{E_2^{'}} \;\Rightarrow\; \left|\nu_{21}^{'}\right|<\left(\frac{E_1^{'}}{E_2^{'}}\right)^{1/2} \qquad (9)$$

The value of $\nu_{12}^{'}$ has a similar restriction; viz.,

$$\left(1-\nu_{12}^{'}\nu_{21}^{'}\right)=1-\left(\frac{E_2^{'}}{E_1^{'}}\nu_{12}^{'}\right)\nu_{12}^{'}>0 \;\Rightarrow\; \left|\nu_{12}^{'}\right|<\left(\frac{E_2^{'}}{E_1^{'}}\right)^{1/2} \qquad (10)$$

Similarly, the values of the remaining four Poisson's ratios, not all independent, have the following restrictions:

$$\left|\nu_{31}^{'}\right| < \left(\frac{E_1^{'}}{E_3^{'}}\right)^{1/2} \;;\; \left|\nu_{13}^{'}\right| < \left(\frac{E_3^{'}}{E_1^{'}}\right)^{1/2} \tag{11}$$

$$\left|\nu_{32}^{'}\right| < \left(\frac{E_2^{'}}{E_3^{'}}\right)^{1/2} \;;\; \left|\nu_{23}^{'}\right| < \left(\frac{E_3^{'}}{E_2^{'}}\right)^{1/2} \tag{12}$$

Finally, Lempriere [45] obtained the following limiting relation:

$$\frac{1}{2} > \frac{1}{2}\left[1 - \left(\nu_{12}^{'}\right)^2\left(\frac{E_2^{'}}{E_1^{'}}\right) - \left(\nu_{13}^{'}\right)^2\left(\frac{E_3^{'}}{E_1^{'}}\right) - \left(\nu_{23}^{'}\right)^2\left(\frac{E_3^{'}}{E_2^{'}}\right)\right] > \nu_{12}^{'}\nu_{31}^{'}\nu_{23}^{'} \tag{13}$$

Equation (13) led Lempriere [45] to conclude that

··· all three Poisson's ratios cannot have large positive values at the same time, as their product must be less than one half. If one is negative, however, no restriction is placed on the other two.

Boulanger and Hayes [6] appear to have been the first to show that Poisson's ratio for an orthotropic material can have no bounds. In such a material, even though the strain energy density is positive definite, Poisson's ratio may assume an arbitrarily large positive value for one pair of orthogonal directions and an arbitrarily small negative value for another pair of orthogonal directions.

Zheng and Chen [76] presented a complete characterization of the admissible sets of Poisson's ratios for orthotropic materials, as well as ones with less internal symmetry. They presented a new perspective on Poisson's ratios of elastic solids. By scaling the Poisson's ratios through the square root of a modulus ratio, the transformed Poisson's ratios were bounded in a closed region which is located inside a cube centered at the origin with a range from -1 to 1.

In a subsequent paper, Ting and Chen [68] have shown that the Poisson's ratios associated with anisotropic elastic materials can have an arbitrarily large positive or negative value under the prerequisite of positive definiteness of strain energy density. This is predicated on the assumption that the material is not subjected to any kinematic constraints such as incompressibility or inextensibility.

8.2 Transversely Isotropic Material Idealizations. For the special case of transverse isotropy, selecting the global x_3-axis normal to the plane of isotropy, $E_1^{'} = E_2^{'} \equiv E_t^{'}$, $E_3^{'} \equiv E_n^{'}$, $\nu_{13}^{'} = \nu_{23}^{'} \equiv \nu_{tn}^{'}$, and $\nu_{12}^{'} = \nu_{21}^{'} \equiv \nu_{tt}^{'}$. Equations (9) and (10), first given by Lempriere [45], reduce to

$$\left|\nu_{tt}^{'}\right| < 1 \quad \Rightarrow \quad -1 < \nu_{tt}^{'} < 1 \tag{14}$$

Equations (11) and (12) reduce to

$$\left|\nu_{nt}^{'}\right| < \left(\frac{E_t^{'}}{E_n^{'}}\right)^{1/2} \;;\; \left|\nu_{tn}^{'}\right| < \left(\frac{E_n^{'}}{E_t^{'}}\right)^{1/2} \tag{15}$$

Thus,

$$-\left(\frac{E_t^{'}}{E_n^{'}}\right)^{1/2} < \nu_{nt}^{'} < \left(\frac{E_t^{'}}{E_n^{'}}\right)^{1/2} \;;\quad -\left(\frac{E_n^{'}}{E_t^{'}}\right)^{1/2} < \nu_{tn}^{'} < \left(\frac{E_n^{'}}{E_t^{'}}\right)^{1/2} \tag{16}$$

For the special case of transverse isotropy, Equation (13) simplifies to

$$\frac{1}{2} > \frac{1}{2}\left[1 - \left(\nu_{tt}^{'}\right)^2 - 2\left(\nu_{tn}^{'}\right)^2\left(\frac{E_n^{'}}{E_t^{'}}\right)\right] > \nu_{tt}^{'}\nu_{nt}^{'}\nu_{tn}^{'} \tag{17}$$

Considering the left inequality in Equation (17) gives the following relation:

$$\left(\nu_{tt}^{'}\right)^2 < 2\left(\nu_{tn}^{'}\right)^2\left(\frac{E_n^{'}}{E_t^{'}}\right) \tag{18}$$

Adopting a more rigorous form of analysis, Pickering [56] noted that, similar to Lempriere [45], the necessary and sufficient condition for the quadratic form $\frac{1}{2}\dot{\boldsymbol{\sigma}}'^{T}\boldsymbol{A}\boldsymbol{\sigma}'$ to be positive definite is that all of the principal minors of **A** should be positive. This leads to the following requirements:

$$E_t^{'} > 0, E_n^{'} > 0, G_{nt} > 0, G_{tt} > 0 \tag{19}$$

Pickering [56] also confirmed that $-1 < \nu_{tt}^{'} < 1$ (recall Eq. (14)). From one of the principal minors of **A**, Pickering [56] obtained the following relation:

$$\frac{E_t^{'}}{E_n^{'}}\left(1-\nu_{tt}^{'}\right) - 2\left(\nu_{tn}^{'}\right)^2 > 0 \tag{20}$$

Pickering [56] noted that, similar to $G_{nt}, E_t^{'}$ and $E_n^{'}$ are independent; only the Poisson's ratios are bounded by the ratio $E_t^{'}/E_n^{'}$.

In commenting on an earlier paper by Barden [3], Raymond [58] stated that, from a zero strain energy function,

$$1 \geq \nu_{tt}^{'} + 2\left(\frac{E_t^{'}}{E_n^{'}}\right)\left(\nu_{nt}^{'}\right)^2 \tag{21}$$

which, except for the case of equality, is identical with Equation (20) given by Pickering [56].

By requiring the dilation to be the same sign as the applied stress, Raymond [58] also stated that $\nu_{nt}^{'} \leq 1/2$, and

$$1 \geq \nu_{tt}^{'} + \left(\frac{E_t^{'}}{E_n^{'}}\right)\nu_{nt}^{'} \tag{22}$$

Finally, Raymond [58] stated that the shear modulus G_{nt} is bounded by the following inequality:

$$G_{nt} \leq \frac{E_n^{'}}{2\nu_{nt}^{'}\left(1+\nu_{tt}^{'}\right)+2\sqrt{\frac{E_n^{'}}{E_t^{'}}\left[1-\left(\nu_{tt}^{'}\right)^2\right]\left[1-\frac{E_t^{'}\left(\nu_{nt}^{'}\right)^2}{E_n^{'}}\right]}} \tag{23}$$

If $E_n^{'} = E_t^{'} \equiv E$ and $\nu_{nt}^{'} = \nu_{tt}^{'} \equiv \nu'$, Equation (23) reduces to

$$G_{nt} \leq \frac{E}{2\nu'(1+\nu') + 2\sqrt{[1-(\nu')^2][1-(\nu')^2]}} = \frac{E}{2(1+\nu')}$$

However, as noted by Raymond [58], since G_{nt} is an independent constant, the conditions $E_n^{'} = E_t^{'}$ and $\nu_{nt}^{'} = \nu_{tt}^{'}$ do not automatically imply isotropy.

8.3 Isotropic Material Idealizations. For their characterization, isotropic materials require the values of two material constants. Traditionally, the "drained" bulk modulus (K') and shear modulus (G), or the "drained" Young's modulus (E') and Poisson's ratio (ν') have been used to characterize isotropic elastic materials. The ratios of these moduli are functions of ν'; viz.,

$$\frac{E'}{G} = 2(1+\nu'); \ \frac{E'}{K'} = 3(1-2\nu'); \ \frac{K'}{G} = \frac{2(1+\nu')}{3(1-2\nu')} \tag{24}$$

Since the strain energy per unit volume $\left(\frac{1}{2}\dot{\boldsymbol{\varepsilon}}^T\mathbf{D}\dot{\boldsymbol{\varepsilon}}\right)$ must positive definite, $E'>0$ and $G>0$. Thus, from the first of Equation (24), $(1+\nu')>0$, implying that $\nu'>-1$. Similarly, since $K'>0$, from the second of Equation (24), $(1-2\nu')>0$, implying that $\nu'<1/2$. Thus, for an isotropic elastic material, the value of Poisson's ratio must fall in the range $-1<\nu'<\frac{1}{2}$ −1. In practice, however, ν' falls into the more limited range of $0 \le \nu' < \frac{1}{2}$ [43; 16].

9. Experimental Determination of Poisson's Ratios for Soils. Although a seemingly straightforward undertaking, the determination of Poisson's ratios for soils is not necessarily a straightforward process. Different approaches are required for isotropic and transversely isotropic material idealizations. These approaches are now briefly reviewed. Additional details pertaining to these aproaches can be found in the cited references.

9.1 Isotropic Material Idealizations. Assuming isotropic elastic response, the simplest way in which to experimentally determine the Poisson's ratio of a soil is to measure the increments in lateral strain $(\Delta\varepsilon_2 = \Delta\varepsilon_3)$ and axial strain $(\Delta\varepsilon_1)$ in an axisymmetric triaxial test. From these measurements, the Poisson's ratio is then computer from the relation $\nu' = -\Delta\varepsilon_3 / \Delta\varepsilon_1$.

In practice, it is not, however, easy to accurately measure $\Delta\varepsilon_3$ in conventional axisymmetric triaxial testing devices. Instead, ν' is determined from $\Delta\varepsilon_1$ and the increment in volumetric strain $(\Delta\varepsilon_\upsilon)$ measured in a drained test. In particular, since $\Delta\varepsilon_2 = \Delta\varepsilon_3$, $\Delta\varepsilon_\upsilon = \Delta\varepsilon_1 + 2\Delta\varepsilon_3$. Thus,

$$\Delta\varepsilon_3 = \frac{1}{2}(\Delta\varepsilon_\upsilon - \Delta\varepsilon_1) \tag{25}$$

Dividing through by $\Delta\varepsilon_1$ gives the desired result; viz.,

$$\nu' = -\frac{\Delta\varepsilon_3}{\Delta\varepsilon_1} = \frac{1}{2}\left(1 - \frac{\Delta\varepsilon_\upsilon}{\Delta\varepsilon_1}\right) \tag{26}$$

To overcome the aforementioned inaccuracies in measuring lateral strains in conventional axisymmetric triaxial devices, certain enhanced devices have been developed that measure both axial and lateral increments in the small strain range (e.g., less than 0.001%). These enhanced devices typically employ proximity transducers (e.g., [28]) or other special sensors (e.g., [31; 48; 26; 29]) that locally measure axial and lateral strains. Values of elastic parameters can also be determined using geophysical field and laboratory methods involving wave propagation. These involve two types of waves, namely surface and body waves. The surface wave of primary interest is the Rayleigh (R) wave. Body waves consist of compression (P) waves that propagate with velocity υ_p, and shear (S) waves that propagate with velocity υ_s.

Equating the relation between G and υ_s [59] with the relation between G and the Young's modulus (E) gives

$$G = p(\upsilon_s)^2 = \frac{E}{2(1+\nu')} \Rightarrow E = 2p(\upsilon_s)^2(1+\nu') \tag{27}$$

The Young's modulus can also be determined from the compressional wave velocity according to

$$E = (\upsilon_p)^2 p \frac{(1+\nu')(1-2\nu')}{1-\nu'} \tag{28}$$

Combining Equations (27) and (28) gives

$$\nu' = \frac{(\upsilon_p)^2 - 2(\upsilon_s)^2}{2\left[(\upsilon_p)^2 - (\upsilon_s)^2\right]} = \frac{(\upsilon_p/\upsilon_s)^2 - 2}{2\left[(\upsilon_p/\upsilon_s)^2 - 1\right]} \qquad (29)$$

9.2 Transversely Isotropic Material Idealizations. The values of the small strain moduli and Poisson's ratios in soils characterized by transversely isotropic elastic material idealizations can be determined from the results of field or laboratory tests. Commonly used field tests include the pressuremeter and in-situ seismic surveys consisting of cross-hole and down-hole techniques. When transversely isotropic shear moduli are determined in the field, the value of G_{nt} is obtained from a down-hole survey in which the shear wave propagates in the vertical direction and the particles move horizontally; in such tests, the shear wave velocity υ_{nt} is measured. By contrast, the value of G_{tt} is determined from a cross-hole survey, in which the shear wave velocity υ_{tt} is measured. Due to the difference in the modes of shear deformation, the results obtained using field tests can, however, be different from laboratory values [73].

Laboratory tests commonly used to determine the small strain moduli and Poisson's ratio values include axisymmetric triaxial, torsional shear, resonant column (RC), and ultrasonic tests. Such tests must be suitably modified in order to determine the values of all five independent parameters associated with a transversely isotropic elastic material idealization. The small-strain probe loadings in such tests are typically applied quasi-statically. The multiaxial body wave measurements in such tests are made using either piezoelectric transducers such as bender elements [7; 52; 46; 40; 28; 51; 21; 53] or other P- and S-wave transducers [4]. Using such laboratory tests allows for either direct measurement [36; 29; 15; 14] or indirect determination [39; 41; 47] of all five independent parameters associated with a transversely isotropic elastic material idealization.

10. Observations of Poisson's Ratios for Soils. Some general observations related to Poisson's ratios determined using the experimental techniques discussed in the previous section are now briefly discussed.

10.1 Isotropic Material Idealizations. The experiments performed by Rowe [60] seemed to show that ν' is indeed isotropic. Hardin [22] noted that, due to the insensitivity of the soil behavior to the value of Poisson's ratio, accurate measurements of ν' from wave propagation experiments were difficult to obtain. However, analysis of his RC tests on Ottawa sand produced constant values of ν' in the range from 0.11 to 0.23. Subsequent experimental findings seem to indicate that, for a given soil at a given void ratio, ν' is essentially constant [13; 43; 26].

Yokota and Konno [75] performed cyclic triaxial (axisymmetric) tests on undisturbed and reconstituted specimens of cohesive soils and alluvial clays. Their investigation showed that, for the cohesive soil, ν' was essentially equal to 0.5, thus reflecting the undrained conditions in these tests. Tests on sandy soils produced values in the range from 0.2 to 0.4. These test results showed a tendency for slightly increasing ν' values with increasing magnitudes of shear strains, whereas the influence of confining pressure was found to be negligible. The latter finding, which was subsequently confirmed for Hostun Sand by El Hosri [13], is consistent with the theoretical findings of Mindlin and Deresiewicz [50]. In a rather limited number of studies [72; 42], ν' for clays has been directly correlated with the plasticity index (I_p).

Whereas Poisson's ratio for a soil at a given void ratio appears to be constant, experimental evidence suggests that ν' increases with increasing void ratio. Re-analysis of unloading branches from numerous tests performed on Ham River sand [12] showed a consistent variation of ν' from 0.20 at a void ratio equal to 0.57 to 0.31 at a void ratio of 0.75 [43]. Lade and Nelson [43] also reported a similar trend for Santa Monica Sand, as did Gu et al. [21] for dry samples of Toyoura, Fujian and Leighton Buzzard sand.

Based on the results of a number of bender elements and extender element tests performed on three dry sands (one fine-grained, one medium-grained, one coarse-grained), Kumar and Madhusudhan [38] noted that the magnitude of ν' decreases with an increase in the magnitude of the effective confining stress and relative density. The effect of the confining stress on ν' was found to be more substantial for the fine-grained sand as compared with the coarse-grained one. For the given sand, at a certain effective confining stress, the magnitude of the ν' was found to decrease almost linearly with an increase in the value of $G_{\max}$.

Hicher [26] noted that for Hostun Sand, ν' appears to slightly depend on the grain size distribution. For poorly-graded samples, $\nu'=0.18$; for well-graded samples, $\nu'=0.32$. Wichtmann and Triantafyllidis [70] performed more than 160 RC tests (with additional P-wave measurements) on samples of a quartz sand that were prepared so as to give 27 different grain size distribution curves from a sieve analyses. From the results of these tests, it was determined that ν' does not depend on the mean grain size (d_{50}) but increases with increasing coefficient of uniformity $C_u = d_{60}/d_{10}$ of the grain size distribution curve.

10.2 Transversely Isotropic Material Idealizations. Test results for Ham River sand obtained by Kuwano and Jardine [40] showed significant scatter in the Poisson ratio values, which underscored the difficulty in precisely measuring radial strains. Values of $\nu_{nt}^{'}$ varied between 0.2 and 0.4, apparently decreasing as p′ increased. Values of $\nu_{tt}^{'}$ were smaller, ranging between 0.05 and 0.20. Kuwano and Jardine [40] concluded that there was no clear difference between the Poisson's ratios of loose and dense specimens. Both $\nu_{nt}^{'}$ and $\nu_{tt}^{'}$ values were slightly larger under anisotropic stress states (K = 0.45).

Yimsiri and Soga [74] determined the values of the five parameters associated with the transversely isotropic elastic material idealization of two natural, overconsolidated stiff clays, namely London clay and Gault clay. Both London clay [27; 33; 15] and Gault clay [55; 47] have been the focus of earlier studies, though these efforts did not provide a complete a complete set of material parameter values. The values of $\nu_{tt}^{'}$ and $\nu_{tn}^{'}$ measured by Yimsiri and Soga [74] exhibited some scatter, and but there was no evident relationship with confining pressure. Consequently, Yimsiri and Soga [74] assumed these Poisson's ratios to be *constant* and *independent* of the confining stress. This is consistent with the earlier findings of Kirkgard and Lade [35] for San Francisco Bay Mud.

Nishimura [54] used an axisymmetric triaxial apparatus with two pairs of bender elements [53] to test saturated sedimentary clays from six different strata located in Japan. In general, the measuring Poisson's ratios at very small strains proved to be very difficult because of the small magnitude of the strains. Values of $\nu_{nt}^{'}$, $\nu_{tn}^{'}$, and $\nu_{tt}^{'}$ were generally the same for all six clays. All three Poisson's ratios did not exhibit any consistent dependence on changes in effective stress. For some of the clays, $\nu_{tt}^{'}$ was negative.

11. Functional Forms Proposed for Poisson's Ratios for Soils. The simplest functional relationship for Poisson's ratio is to assume it to be constant. This has been done in both isotropic [13; 43; 26] and transversely isotropic [35; 11; 40; 74; 54] material idealizations. Since experimental evidence is not, however, universally supportive of such an assumption, it is instructive to briefly review some functional forms that assume a variable Poisson's ratio.

11.1 Isotropic Material Idealizations. Based on the results of tests performed on three dry uniform sands (Toyoura, Fujian and Leighton Buzzard), Gu et al. [21] found that at a given pressure, ν' increases with increases in void ratio (*e*). This findings is consistent with the earlier results of Daramola [12] and Lade and Nelson [43]. At the same void ratio, ν' was found to decrease with increases in confining stress. Gu et al. [21] thus proposed the following general functional form for ν':

$$\nu' = Ae^{-x}\left(\frac{\sigma'}{p_{ref}}\right)^n \qquad (30)$$

where *n* is another model parameter, σ' is the effective confining stress, and p_{ref} is again a reference stress (taken equal to 98 kPa). The following functions were determined for Toyoura, Fujian, and Leighton Buzzard sand:

$$\nu' = 0.254e^{-0.29}\left(\frac{\sigma'}{p_{ref}}\right)^{-0.09};\ \nu' = 0.305e^{-0.56}\left(\frac{\sigma'}{p_{ref}}\right)^{-0.09};\ \nu' = 0.275e^{-0.47}\left(\frac{\sigma'}{p_{ref}}\right)^{-0.05} \qquad (31)$$

respectively.

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11.2 Transversely Isotropic Material Idealizations. The results of axisymmetric triaxial tests performed on various uncemented sands and gravels [62; 32; 29] demonstrated that such soils are inherently transversely isotropic. Based on these results, a hypoelastic constitutive model was proposed by Tatsuoka and Kohata [62] and then subsequently refined by Hoque et al. [30], Jiang et al. [32], Hoque and Tatsuoka [29], and Tatsuoka et al. [63]. This model is based on the assumptions of Hardin [22] and Hardin and Bladford [24] that the Young's modulus (E_i) associated with a particular coordinate direction is a unique function of the normal stress acting in this direction, and is independent of the other two orthogonal directions.

Under axisymmetric triaxial conditions with the x_1 direction oriented normal to the plane of isotropy, $\sigma_{22}' = \sigma_{33}'$. The Young's moduli are then assumed to have the following "power" form:

$$E_n' = E_1^* F(e) \left(\frac{\sigma_{11}'}{\sigma_{ref}'}\right)^m \quad (32)$$

$$E_t' = E_1^* R_E F(e) \left(\frac{\sigma_{33}'}{\sigma_{ref}'}\right)^m \quad (33)$$

where *m* is a model parameter, and $F(e)$ is the following function of the void ratio *e* [29]:

$$F(e) = \frac{(2.17-e)^2}{1+e} \quad (34)$$

which was proposed for sands with rounded grains by Hardin and Richart [25] and Hardin and Black [23].

In Equations (32) and (33),

$$E_1^* = \frac{(E_n')_{ref}}{F(e_{ref})} \quad (35)$$

where $(E_n')_{ref}$ is the value of E_n' for $\sigma_{11}' = \sigma_{ref}'$, and $F(e_{ref})$ is the value of $F(e)$ for $e = e_{ref}$; i.e., the void ratio associated with $\sigma_{11}' = \sigma_{ref}'$. The quantity σ_{ref}' is a reference stress that has the same units as $\sigma_{11}', \sigma_{22}'$, and σ_{33}'. In the past, σ_{ref}' has typically been set equal to the atmospheric pressure (P_a). Finally, in Equation (33), $R_E = E_t' / E_n'$ represents the degree of inherent anisotropy ($R_E = 1.0$ implies an inherently isotropic material).

From Equations (32) and (33),

$$n = \frac{E_t'}{E_n'} R_E (R_\sigma)^{1/m} \quad (36)$$

where $R_\sigma = \sigma_{11}' / \sigma_{33}' = \sigma_n' / \sigma_t'$, which represents the stress-induced anisotropy.

Assuming symmetry of **A**, $\nu_{tn}' = \nu_{nt}' R_E (E_t'/E_n')$. Substituting Equations (32) and (33) into this expression gives

$$\nu_{tn}' = \nu_{nt}' R_E \left(\frac{1}{R_\sigma}\right)^m \quad (37)$$

Although Equation (37) is consistent with Equations (32) and (33), it does not give *explicit* expressions for either ν_{tn}' or ν_{nt}'. To overcome this shortcoming, Hoque and Tatsuoka [29] developed alternate functional forms for ν_{tn}' and ν_{nt}'. They noted that, based on the results of axisymmetric triaxial tests performed on different sands at various stress states in the range $0.5 \le R_\sigma \le 2.0$, the Poisson's ratio ν_{nt}' did not vary greatly. Within the range of pressures investigated, the value of ν_{nt}' was not sensitive to the change in σ_n' or σ_t' at a fixed ratio R_σ. This Poisson's ratio did, however, gradually increase with R_σ.

To account for this fact and to include the effect of inherent anisotropy (via R_E) while also satisfying the relation $\nu'_{tn}/E'_t = \nu'_{nt}/E'_n$ that comes from the symmetry of **A**, Hoque and Tatsuoka [29] proposed the following functional forms:

$$\nu'_{nt} = \nu_0 \sqrt{\frac{1}{R_E}} (R_\sigma)^{m/2}; \nu'_{tn} = \nu_0 \sqrt{R_E} \left(\frac{1}{R_\sigma}\right)^{m/2} \quad (38)$$

Regarding the Poisson's ratio ν'_{tt}, Hoque and Tatsuoka [29] proposed that $\nu'_{tn} = \nu_0$. Then, for isotropic materials (i.e., $R_E = 1.0$) subjected to isotropic stress states, $\nu'_{nt} = \nu'_{tn} = \nu'_{tt} = \nu_0$. Clearly, this choice of ν'_{tt} is somewhat arbitrary.

In summary, although the model proposed by Tatsuoka and co-workers is arguably the most advanced transversely isotropic elastic formulation for soils, it still possesses several shortcomings. Firstly, it only predicts truly transversely isotropic elastic response under axisymmetric triaxial conditions. Thus, it is not a general transversely isotropic elastic model. Secondly, its treatment of Poisson's ratios in general, and ν'_{tt} in particular, is somewhat arbitrary.

12. Conclusions. Some key issues associated with the seemingly straightforward task of determining Poisson ratio values for soils have been presented in this paper. Although some of these issues have been previously discussed in other publications, the focus in these earlier works has been on isotropic elastic material idealizations. The emphasis in this paper has thus been on transversely isotropic elastic idealizations. As in the case of isotropic formulations, there is no consensus on the proper functional representation of Poisson's ratios for transversely isotropic elastic idealizations. This is, in part, due to the difficulties in accurately measuring the very small lateral strains generated by axial loading in the elastic range. Such inaccuracies in measurement complicate the development of definitive conclusions related to Poisson's ratios for soils. They also point to the need for performing additional investigations that measure Poisson's ratios for transversely isotropic soils, especially for cohesive soils.

References

1 Arroyo, M., Muir Wood, D., and Greening, P. D. Source near-field effects and pulse tests in soil samples. Geotechnique- 2003.Vol. 53.№3.P.337–345.

2 Atkinson, J. H. Anisotropic elastic deformations in laboratory tests on undisturbed London clay. Geotechnique -1975. Vol. 25.№2.P. 357–374,

3 Barden, L. Stresses and displacements in a cross-anisotropic soil. Geotechnique, 13(3):198–210, 1963.

4 Bellotti, L., Jamiokowski, M., Lo Presti, D. C. F., and O'Neill, D. A. Anisotropy of small strain stiffness in Tocino sand. Geotechnique -1996 Vol. 46.№1.P.115–131.

5 Boulanger, P. H. and Hayes, M. On young's modulus for anisotropic media// Journal of Applied Mechanics. -ASME - 1995. Vol. 62.-P.819–820.

6 Boulanger, P. H. and Hayes, M. Poisson's ratio for orthorhombic materials. Journal of Elasticity.- 1998. Vol. 50.№1.-P.87–89.

7 Brignoli, E. G .M., Gotti, M., and Stokoe, K. H. Measurement of shear waves in laboratory specimens by means of piezoelectric transducers. Geotechnical Testing Journal- 1996. Vol. 19.№4.-P.384–397.

8 Casagrande, A. and Carrillo, N. Shear failure of anisotropic materials. Proceedings of the Boston Society of Civil Engineers.- 1944. Vol. 31.-P.74–87.

9 Cazzani, A. and Rovati, M. Extrema of young's modulus for cubic and transversely isotropic solids. International Journal of Solids and Structures.-2003. Vol. 40.-P.1713–1744.

10 Cazzani, A. and Rovati, M. Extrema of young's modulus for elastic solids with tetragonal symmetry. International Journal of Solids and Structures. -2005. Vol. 42.-P.1713–1744.

11 Dakoulas, P. and Yu, S. Stress-dependency of elastic moduli for cross-anisotropic soils. Geotechnique.- 1995. Vol. 45.№2.-P.325–332.

12 Daramola, O. On estimating k0 for overconsolidated granular soils. Geotechnique.- 1980. 30.№3.-P.310–313.

13 El Hosri M. S. Contribution ´a l' ´etude des propri`etes m´ecanique des materiaux; PhD thesis. University of Paris, France, 1984.

14 Ezaoui, A. and Di Benedetto, H. Experimental measurements of the global anisotropic elastic behavior of dry Hostun sand during triaxial tests, and the effect of sample preparation. Geotechnique. -2009.Vol. 59.№7.-P.621–635.

15 Gasparre, A., Nishimura, S., Anh-Minh, N., Coop, M. R., and Jardine, R. J. The stiffness of natural London clay. Geotechnique.- 2007. Vol. 57.№1.P.33–47.

16 Gercek, H. Poisson's ratio values for rocks. International Journal of Rock Mechanics and Mineral Science.- 2007. Vol. 44.№1.-P.1–13.

17 Gerrard, C. M. Background to mathematical modeling in Geomechanics: the role of fabric and stress history. In Gudehus, G., editor, Finite Elements in //Geomechanics, chapter 2, pages John Wiley and Sons. – London; 1977. –P. 33–120.

18 Gibson, R. E. The analytical method in soil mechanics. Geotechnique. -1974. Vol.24.№2.-P.115–140.

19 Graham, J. and Houlsby, T. Anisotropic elasticity of a natural clay. Geotechnique - 1983. Vol. 33.№2.-P.165–180.

20 Greaves, G. N., Greer, A. L., Lakes, R. S., and Rouxel, T. Poisson's ratio and modern materials. Nature Materials. -2011. Vol.10.№823.-P.823–837.

21 Gu, X., Yang, J., and Huang, M. Laboratory measurements of small strain properties of dry sands by bender element. Soils and Foundations.- 2013. Vol.53.№5.-P.735–745.

22 Hardin, B. O. The nature of stress-strain behavior for soils. In Proceedings of the Specialty Conference on Earthquake Engineering and Soil Dynamics, pages, Pasadena, CA, ASCE. -1978.-P. 3–90.

23 Hardin, B. O. and Black, W. L. Sand stiffness under various triaxial stresses. Journal of the Soil Mechanics and Foundations Division, ASCE, 92(SM2) 1966.-P. 27–42.

24 Hardin, B. O. and Bladford, G. E. Elasticity of particulate materials. Journal of Geotechnical Engineering, ASCE.- 1969. Vol.89.№6.-P.788–805.

25 Hardin, B. O. and Richart, F. E. Elastic wave velocities in granular soils. Journal of the Soil Mechanics and Foundations Division, ASCE, 89(SM1).- 1968.-P.33–65.

26 Hicher, P. Y. Elastic properties of soils. Journal of Geotechnical Engineering, ASCE.- 1996. Vol.122.№8.-P.641–648.

27 Hight, D. W., Bennell, J. D., Chana, B., Davis, P. D., Jardine, R. J., and Porovic, J. E. Wave velocity and stiffness measurements of the Crag and Lower London Tertiaries at Sizewell. Geotechnique. 1997. Vol.47.№3.-P.451–474.

28 Hird, C. and Yung, P. The use of proximity transducers for local strain measurements in triaxial tests. Geotechnical Testing Journal. -1989. Vol.12.№4.-P.292–300.

29 Hoque, E. and Tatsuoka, F. Anisotropy in the elastic deformation of materials. Soils and Foundations.- 1998. Vol. 38.№1.P.163–179.

30 Hoque, E., Tatsuoka, F., and Sato, T. Measuring anisotropic properties of sands using a large triaxial specimen. Geotechnical Testing Journal. - 1996. Vol.19.№4.-P.411–420.

31 Jardine, R. J., Symes, M. J., and Burland, J. B. The measurement of soil stiffness in the triaxial apparatus. Geotechnique. -1984.Vol.34.-P.3.-P.323–340.

32 Jiang, G-L., Tatsuoka, F., Flora, A., and Koseki, J. Inherent and stress-induced anisotropy in very small strain stiffness of a sandy gravel. Geotechnique. - 1997. Vol.47.№3.-P.509–521.

33 Joviˇci´c, V. and Coop, M. R. The measurement of stiffness anisotropy in clays with bender element tests in the triaxial apparatus. Geotechnical Testing Journal. -1998. Vol.21.№1.-P.3–10.

34 Kaliakin, V. N. Anisotropic elasticity for soils: A synthesis of some key issues. Bulletin of L.N. Gumilyov Eurasian National University, Technical Science and Technology Series. - 2019. Vol.127.№2.-P.49–63.

35 Kirkgard, M. M. and Lade, P. V. Anisotropy of normally consolidated San Francisco bay mud. ASTM Geotechnical Testing Journal.- 1991.Vol.14.№3.-P.231–246.

36 Kokusho, T. Cyclic triaxial test of dynamic soil properties for wide strain range. Soils and Foundations.- 1980. Vol.20.№2.-P.45–60.

37Krizek, R. J. Fabric effects on strength and deformation of kaolin clay. In Proceedings of the 9th International Conference on Soil Mechanics and Foundation Engineering, pages, Tokyo, Japan, -1977.-P. 169–176.

38 Kumar, J. and Madhusudhan, B. N. Effect of relative density and confining pressure on Poisson ratio from bender-extender element tests. Geotechnique.- 2010. Vol.60.№7.-P.561–567.

39 Kuwano, R. and Jardine, R. J. Stiffness measurements in a stress-path cell. In et al., R. J. Jardine, editor, Pre-Failure Deformation Behaviour of Geo-materials, pages, London, UK, Thomas Telford.- 1998.-P. 391–394.

40 Kuwano, R. and Jardine, R. J. On the applicability of cross-anisotropic elasticity to granular materials at very small strains. Geotechnique.- 2002. Vol.52.№10.-P.727–749.

41 Kuwano, R., Connolly, T. M., and Jardine, R. J. Anisotropic stiffness measurements in a stress-path triaxial cell. Geotechnical Testing Journal.- 2000. Vol.23. №2.-P.141–157.

42Lade, P. V. Calibration of the single hardening constitutive model for clays. In Proceedings of the 11th International Conference of the Association for Computer Methods and Advances in Geomechanics (IACMAG), pages CD–ROM, Turin, Italy, 2005.

43 Lade, P. V. and Nelson, R. B. Modelling the elastic behaviour of granular materials. International Journal for Numerical and Analytical Methods in Geomechanics.- 1987. Vol.11.-P. 521–542,

44 Lekhnitskii, S. G. Theory of Elasticity of an Anisotropic Body. Mir Publishers, Moscow, Russia, 1981.

45 Lempriere, B. M. Poisson's ratio in orthotropic materials. American Institute of Aeronautics and Astronautics Journal. - 1968. Vol.6. №11.-P. 2226–2227.

46 Lings, M. L. and Greening, P. D. A novel bender/extender element for soil testing. Geotechnique.- 2001. Vol.51. №8.-P.713–717.

47 Lings, M. L., Pennington, D. S., and Nash, D. F. T. Anisotropic stiffness parameters and their measurement in a stiff natural clay. Geotechnique. -2000. Vol.50. №2.-P. 109–125.

48 Lo Presti, D. C. F., Pallara, O., and Impavido, M. Small strain measurements during triaxial tests: Many problems, some solutions. In Shibuya, S., Mitachi, T., and Miura, S., editors, International Conference on Pre-failure Deformation Characteristics of Geomaterials, pages, Hokkaido University, Sapporo, Japan, Balkema, Rotterdam.- 1994.-P. 11–16

49 Love, A. E. H. A Treatise on the Mathematical Theory of Elasticity. Cambridge Univ. Press, Cambridge, UK, 4th edition, 1927.

50 Mindlin, R. D. and Deresiewicz, H. Elastic spheres in contact under varying oblique forces. Journal of Applied Mechanics, ASME. - 1953. Vol.20. №3.-P.327–344.

51 Murillo, C., Sharifipour, M., Caicedo, B., Thorel, L., and Dano, C. Elastic parameters of intermediate soils based on bender-extender elements pulse tests. Soils and Foundations.- 2011. Vol.51. №4.-P.637–649.

52 Nakagawa, K., Soga, K., and Mitchell, J.K. Observation of Biot compressional wave of the second kind in granular soils. Geotechnique.- 1997. Vol.47. №1.-P.133–147.

53 Nishimura, S. Assessment of anisotropic elastic parameters of saturated clay measured in triaxial apparatus: Appraisal of techniques and derivation procedures. Soil and Foundations. - 2014. Vol.54. №3.-P.364–376.

54 Nishimura, S. Cross-anisotropic deformation characteristics of natural sedimentary clays. Geotechnique e.- 2014. Vol.64. №12.-P.981–996.

55 Pennington, D. S., Nash, D. F. T., and Lings, M. L. Anisotropy of g0 shear stiffness in Gault clay. Geotechnique. -1997. Vol.47. №3.-P.391–398.

56 Pickering, D. J. Anisotropic elastic parameters for soil. Geotechnique. - 1970. Vol.20. №3.-P.271–276.

57 Poisson, S. D. M´e moire sur l´´e quilibre et le mouvement des corps ´e lastiques. Mem. de l'Acad. Paris, -P.8,-1829.

58 Raymond, G. P. Discussion: Stresses and displacements in a cross-anisotropic soil, by L. Barden. Geotechnique. -1970. Vol.20. №4.-P.456–458.

59 Richart, F. E., Hall, J. R., and Woods, R. D. Vibrations of Soils and Foundations. Prentice Hall, Englewood Cliffs, NJ, 1st edition, 1970.

60 Rowe, P. W. Theoretical meaning and observed values of deformation parameters for soil. In Parry, R. H. G., editor, Proceedings of the Roscoe Memorial Symposium on Stress-Strain Behavior of Soils, pages, Henley-on-Thames, UK, G.T. Foulis.-1971.-P. 143–194.

61 Sivakumar, V., Doran, I. G., Graham, J., and Johnson, A. The effect of anisotropic elasticity on the yielding characteristics of overconsolidated natural clay. Canadian Geotechnical Journal.- 2001. Vol.38. №1.-P.125–137.

62 Tatsuoka, F. and Kohata, Y. Stiffness of hard soils and soft rocks in engineering applications. In Shibuya, et al., editor, Proceedings of the International Symposium on Pre-Failure Deformation of Geomaterials (IS-Hokkaido), pages, Hokkaido, Japan, Balkema.- 1995.-P. 947–1063.

63 Tatsuoka, F., Uchimura, T., Hayano, K., Di Benedetto, H., Koseki, J., and Siddique, M. S. A. Time-dependent deformation characteristics of stiff geomaterials in engineering practice. In Proceedings of the Second International Conference on Pre-Failure Deformation Characteristics of Geomaterials, pages, Torino, Italy, Balkema.- 2001.-P. 1161–1262.

64 Timoshenko, S. P. History of Strength of Materials. Dover Publications, Inc., New York, NY, 1983.

65 Ting, T. C. T. Positive definiteness of anisotropic elastic constants. Mathematics and Mechanics of Solids. - 1996. Vol.1 . №3.-P.301–314.

66 Ting, T. C. T. The stationary values of Young's modulus for monoclinic and triclinic materials. Journal Mechanics.- 2005.Vol.21. №4.-P.249–253.

67 Ting, T. C. T. Explicit expression of the stationary values of Young's modulus and the shear modulus for anisotropic elastic materials. Journal Mechanics. -2005. Vol.21. №4.-P.255–266.

68 Ting, T. C. T. and Chen, T. Poisson's Ratio for Anisotropic Elastic Materials Can Have No Bounds. Quarterly Journal of Mechanics and Applied Mathematics. -2005. Vol.58. №1.-P.73–82.

69 Todhunter, I. and Pearson, K. A History of the Theory of Elasticity and of the Strength of Materials: from Galilei to the Present Time. Cambridge University Press, Cambridge, U.K., 1993.

70 Wichtmann, T. and Triantafyllidis, T. On the influence of the grain size distribution curve on P-wave velocity, constrained elastic modulus Mmax and Poisson's ratio of quartz sands. Soil Dynamics and Earthquake Engineering.- 2010.Vol.30. №8.-P.757–766.

71 Wong, P. K. K. and Mitchell, R. J. Yielding and plastic flow of sensitive cemented clays. Geotechnique. -1975.Vol.25. №4.-P.763–782.

72 Wroth, C. P. In-situ measurement of initial stresses and deformation characteristics. In Proceedings of the ASCE Specialty Conference on In-Situ Measurement of Soil properties, pages, Raleigh, NC, ASCE.- 1975.-P. 180–230.

73 Yimsiri, S. and Soga, K. Application of micromechanics model to study anisotropy of soils at small strains. Soils and Foundations.- 2002.Vol.42. №5.-P.15–26.

74 Yimsiri, S. and Soga, K. Cross-anisotropic elastic parameters of two natural stiff clays. Geotechnique. -2011.Vol.61. №9.-P.809–814.

75 Yokota, K. and Konno, M. Dynamic Poisson's Ratio of Soil. In Proceedings of the 7th World Conference on Earthquake Engineering, pages Istanbul, Turkey, 1980.-P. 475–478.

76 Zheng, Q.-S. and Chen, T. New perspective on poisson's ratios of elastic solids. Acta Mechanica. -2001. Vol.150. №3.-P.191–195. 09.

References

1 Arroyo, M., Muir Wood, D., and Greening, P. D. Source near-field effects and pulse tests in soil samples. Geotechnique- 2003.Vol. 53. №3.P.337–345.

2 Atkinson, J. H. Anisotropic elastic deformations in laboratory tests on undisturbed London clay. Geotechnique -1975. Vol. 25. №2.P. 357–374,

3 Barden, L. Stresses and displacements in a cross-anisotropic soil. Geotechnique, 13(3):198–210, 1963.

4 Bellotti, L., Jamiokowski, M., Lo Presti, D. C. F., and O'Neill, D. A. Anisotropy of small strain stiffness in Tocino sand. Geotechnique -1996 Vol. 46. №1.P.115–131.

5 Boulanger, P. H. and Hayes, M. On young's modulus for anisotropic media// Journal of Applied Mechanics. -ASME - 1995. Vol. 62.-P.819–820.

6 Boulanger, P. H. and Hayes, M. Poisson's ratio for orthorhombic materials. Journal of Elasticity.- 1998. Vol. 50. №1.-P.87–89.

7 Brignoli, E. G .M., Gotti, M., and Stokoe, K. H. Measurement of shear waves in laboratory specimens by means of piezoelectric transducers. Geotechnical Testing Journal- 1996. Vol. 19. №4.-P.384–397.

8 Casagrande, A. and Carrillo, N. Shear failure of anisotropic materials. Proceedings of the Boston Society of Civil Engineers.- 1944. Vol. 31.-P.74–87.

9 Cazzani, A. and Rovati, M. Extrema of young's modulus for cubic and transversely isotropic solids. International Journal of Solids and Structures.-2003. Vol. 40.-P.1713–1744.

10 Cazzani, A. and Rovati, M. Extrema of young's modulus for elastic solids with tetragonal symmetry. International Journal of Solids and Structures. -2005. Vol. 42.-P.1713–1744.

11 Dakoulas, P. and Yu, S. Stress-dependency of elastic moduli for cross-anisotropic soils. Geotechnique.- 1995. Vol. 45. №2.-P.325–332.

12 Daramola, O. On estimating k0 for overconsolidated granular soils. Geotechnique.- 1980. 30. №3.-P.310–313.

13 El Hosri M. S. Contribution ´a l' ´etude des propri`etes m´ecanique des materiaux; PhD thesis. University of Paris, France, 1984.

14 Ezaoui, A. and Di Benedetto, H. Experimental measurements of the global anisotropic elastic behavior of dry Hostun sand during triaxial tests, and the effect of sample preparation. Geotechnique. -2009.Vol. 59. №7.-P.621–635.

15 Gasparre, A., Nishimura, S., Anh-Minh, N., Coop, M. R., and Jardine, R. J. The stiffness of natural London clay. Geotechnique.- 2007. Vol. 57. №1.P.33–47.

16 Gercek, H. Poisson's ratio values for rocks. International Journal of Rock Mechanics and Mineral Science.- 2007. Vol. 44. №1.-P.1–13.

17 Gerrard, C. M. Background to mathematical modeling in Geomechanics: the role of fabric and stress history. In Gudehus, G., editor, Finite Elements in //Geomechanics, chapter 2, pages John Wiley and Sons. – London; 1977. –P. 33–120.

18 Gibson, R. E. The analytical method in soil mechanics. Geotechnique. -1974. Vol.24.№2.-P.115–140.

19 Graham, J. and Houlsby, T. Anisotropic elasticity of a natural clay. Geotechnique - 1983. Vol. 33.№2.-P.165–180.

20 Greaves, G. N., Greer, A. L., Lakes, R. S., and Rouxel, T. Poisson's ratio and modern materials. Nature Materials. -2011. Vol.10.№823.-P.823–837.

21 Gu, X., Yang, J., and Huang, M. Laboratory measurements of small strain properties of dry sands by bender element. Soils and Foundations.- 2013. Vol.53.№5.-P.735–745.

22 Hardin, B. O. The nature of stress-strain behavior for soils. In Proceedings of the Specialty Conference on Earthquake Engineering and Soil Dynamics, pages, Pasadena, CA, ASCE. -1978.-P. 3–90.

23 Hardin, B. O. and Black, W. L. Sand stiffness under various triaxial stresses. Journal of the Soil Mechanics and Foundations Division, ASCE, 92(SM2) 1966.-P. 27–42.

24 Hardin, B. O. and Bladford, G. E. Elasticity of particulate materials. Journal of Geotechnical Engineering, ASCE.- 1969. Vol.89.№6.-P.788–805.

25 Hardin, B. O. and Richart, F. E. Elastic wave velocities in granular soils. Journal of the Soil Mechanics and Foundations Division, ASCE, 89(SM1).- 1968.-P.33–65.

26 Hicher, P. Y. Elastic properties of soils. Journal of Geotechnical Engineering, ASCE.- 1996. Vol.122.№8.-P.641–648.

27 Hight, D. W., Bennell, J. D., Chana, B., Davis, P. D., Jardine, R. J., and Porovic, J. E. Wave velocity and stiffness measurements of the Crag and Lower London Tertiaries at Sizewell. Geotechnique. 1997. Vol.47.№3.-P.451–474.

28 Hird, C. and Yung, P. The use of proximity transducers for local strain measurements in triaxial tests. Geotechnical Testing Journal. -1989. Vol.12.№4.-P.292–300.

29 Hoque, E. and Tatsuoka, F. Anisotropy in the elastic deformation of materials. Soils and Foundations.- 1998. Vol. 38.№1.P.163–179.

30 Hoque, E., Tatsuoka, F., and Sato, T. Measuring anisotropic properties of sands using a large triaxial specimen. Geotechnical Testing Journal. - 1996. Vol.19.№4.-P.411–420.

31 Jardine, R. J., Symes, M. J., and Burland, J. B. The measurement of soil stiffness in the triaxial apparatus. Geotechnique. -1984.Vol.34.-P.3.-P.323–340.

32 Jiang, G-L., Tatsuoka, F., Flora, A., and Koseki, J. Inherent and stress-induced anisotropy in very small strain stiffness of a sandy gravel. Geotechnique. - 1997. Vol.47.№3.-P.509–521.

33 Joviˇci´c, V. and Coop, M. R. The measurement of stiffness anisotropy in clays with bender element tests in the triaxial apparatus. Geotechnical Testing Journal. -1998. Vol.21.№1.-P.3–10.

34 Kaliakin, V. N. Anisotropic elasticity for soils: A synthesis of some key issues. Bulletin of L.N. Gumilyov Eurasian National University, Technical Science and Technology Series. - 2019. Vol.127.№2.-P.49–63.

35 Kirkgard, M. M. and Lade, P. V. Anisotropy of normally consolidated San Francisco bay mud. ASTM Geotechnical Testing Journal.- 1991.Vol.14.№3.-P.231–246.

36 Kokusho, T. Cyclic triaxial test of dynamic soil properties for wide strain range. Soils and Foundations.- 1980. Vol.20.№2.-P.45–60.

37Krizek, R. J. Fabric effects on strength and deformation of kaolin clay. In Proceedings of the 9th International Conference on Soil Mechanics and Foundation Engineering, pages, Tokyo, Japan, -1977.-P. 169–176.

38 Kumar, J. and Madhusudhan, B. N. Effect of relative density and confining pressure on Poisson ratio from bender-extender element tests. Geotechnique.- 2010. Vol.60.№7.-P.561–567.

39 Kuwano, R. and Jardine, R. J. Stiffness measurements in a stress-path cell. In et al., R. J. Jardine, editor, Pre-Failure Deformation Behaviour of Geo-materials, pages, London, UK, Thomas Telford.- 1998.-P. 391–394.

40 Kuwano, R. and Jardine, R. J. On the applicability of cross-anisotropic elasticity to granular materials at very small strains. Geotechnique.- 2002. Vol.52.№10.-P.727–749.

41 Kuwano, R., Connolly, T. M., and Jardine, R. J. Anisotropic stiffness measurements in a stress-path triaxial cell. Geotechnical Testing Journal.- 2000. Vol.23.№2.-P.141–157.

42Lade, P. V. Calibration of the single hardening constitutive model for clays. In Proceedings of the 11th International Conference of the Association for Computer Methods and Advances in Geomechanics (IACMAG), pages CD–ROM, Turin, Italy, 2005.

43 Lade, P. V. and Nelson, R. B. Modelling the elastic behaviour of granular materials. International Journal for Numerical and Analytical Methods in Geomechanics.- 1987. Vol.11.-P. 521–542,

44 Lekhnitskii, S. G. Theory of Elasticity of an Anisotropic Body. Mir Publishers, Moscow, Russia, 1981.

45 Lempriere, B. M. Poisson's ratio in orthotropic materials. American Institute of Aeronautics and Astronautics Journal. - 1968. Vol.6.№11.-P. 2226–2227.

46 Lings, M. L. and Greening, P. D. A novel bender/extender element for soil testing. Geotechnique.- 2001. Vol.51.№8.-P.713–717.

47 Lings, M. L., Pennington, D. S., and Nash, D. F. T. Anisotropic stiffness parameters and their measurement in a stiff natural clay. Geotechnique. -2000. Vol.50.№2.-P. 109–125.

48 Lo Presti, D. C. F., Pallara, O., and Impavido, M. Small strain measurements during triaxial tests: Many problems, some solutions. In Shibuya, S., Mitachi, T., and Miura, S., editors, International Conference on Pre-failure Deformation Characteristics of Geomaterials, pages, Hokkaido University, Sapporo, Japan, Balkema, Rotterdam.- 1994.-P. 11–16

49 Love, A. E. H. A Treatise on the Mathematical Theory of Elasticity. Cambridge Univ. Press, Cambridge, UK, 4th edition, 1927.

50 Mindlin, R. D. and Deresiewicz, H. Elastic spheres in contact under varying oblique forces. Journal of Applied Mechanics, ASME. - 1953. Vol.20.№3.-P.327–344.

51 Murillo, C., Sharifipour, M., Caicedo, B., Thorel, L., and Dano, C. Elastic parameters of intermediate soils based on bender-extender elements pulse tests. Soils and Foundations.- 2011. Vol.51.№4.-P.637–649.

52 Nakagawa, K., Soga, K., and Mitchell, J.K. Observation of Biot compressional wave of the second kind in granular soils. Geotechnique.- 1997. Vol.47.№1.-P.133–147.

53 Nishimura, S. Assessment of anisotropic elastic parameters of saturated clay measured in triaxial apparatus: Appraisal of techniques and derivation procedures. Soil and Foundations. - 2014. Vol.54.№3.-P.364–376.

54 Nishimura, S. Cross-anisotropic deformation characteristics of natural sedimentary clays. Geotechnique e.- 2014. Vol.64.№12.-P.981–996.

55 Pennington, D. S., Nash, D. F. T., and Lings, M. L. Anisotropy of g0 shear stiffness in Gault clay. Geotechnique. -1997. Vol.47.№3.-P.391–398.

56 Pickering, D. J. Anisotropic elastic parameters for soil. Geotechnique. - 1970. Vol.20.№3.-P.271–276.

57 Poisson, S. D. M´e moire sur l´´e quilibre et le mouvement des corps ´e lastiques. Mem. de l'Acad. Paris, -P.8,-1829.

58 Raymond, G. P. Discussion: Stresses and displacements in a cross-anisotropic soil, by L. Barden. Geotechnique. -1970. Vol.20.№4.-P.456–458.

59 Richart, F. E., Hall, J. R., and Woods, R. D. Vibrations of Soils and Foundations. Prentice Hall, Englewood Cliffs, NJ, 1st edition, 1970.

60 Rowe, P. W. Theoretical meaning and observed values of deformation parameters for soil. In Parry, R. H. G., editor, Proceedings of the Roscoe Memorial Symposium on Stress-Strain Behavior of Soils, pages, Henley-on-Thames, UK, G.T. Foulis.-1971.-P. 143–194.

61 Sivakumar, V., Doran, I. G., Graham, J., and Johnson, A. The effect of anisotropic elasticity on the yielding characteristics of overconsolidated natural clay. Canadian Geotechnical Journal.- 2001. Vol.38.№1.-P.125–137.

62 Tatsuoka, F. and Kohata, Y. Stiffness of hard soils and soft rocks in engineering applications. In Shibuya, et al., editor, Proceedings of the International Symposium on Pre-Failure Deformation of Geomaterials (IS-Hokkaido), pages, Hokkaido, Japan, Balkema.- 1995.-P. 947–1063.

63 Tatsuoka, F., Uchimura, T., Hayano, K., Di Benedetto, H., Koseki, J., and Siddique, M. S. A. Time-dependent deformation characteristics of stiff geomaterials in engineering practice. In Proceedings of the Second International Conference on Pre-Failure Deformation Characteristics of Geomaterials, pages, Torino, Italy, Balkema.- 2001.-P. 1161–1262.

64 Timoshenko, S. P. History of Strength of Materials. Dover Publications, Inc., New York, NY, 1983.

65 Ting, T. C. T. Positive definiteness of anisotropic elastic constants. Mathematics and Mechanics of Solids. - 1996. Vol.1 .№3.-P.301–314.

66 Ting, T. C. T. The stationary values of Young's modulus for monoclinic and triclinic materials. Journal Mechanics.- 2005.Vol.21.№4.-P.249–253.

67 Ting, T. C. T. Explicit expression of the stationary values of Young's modulus and the shear modulus for anisotropic elastic materials. Journal Mechanics. -2005. Vol.21.№4.-P.255–266.

68 Ting, T. C. T. and Chen, T. Poisson's Ratio for Anisotropic Elastic Materials Can Have No Bounds. Quarterly Journal of Mechanics and Applied Mathematics. -2005. Vol.58.№1.-P.73–82.

69 Todhunter, I. and Pearson, K. A History of the Theory of Elasticity and of the Strength of Materials: from Galilei to the Present Time. Cambridge University Press, Cambridge, U.K., 1993.

70 Wichtmann, T. and Triantafyllidis, T. On the influence of the grain size distribution curve on P-wave velocity, constrained elastic modulus Mmax and Poisson's ratio of quartz sands. Soil Dynamics and Earthquake Engineering.- 2010.Vol.30.№8.-P.757–766.

71 Wong, P. K. K. and Mitchell, R. J. Yielding and plastic flow of sensitive cemented clays. Geotechnique. -1975.Vol.25.№4.-P.763–782.

72 Wroth, C. P. In-situ measurement of initial stresses and deformation characteristics. In Proceedings of the ASCE Specialty Conference on In-Situ Measurement of Soil properties, pages, Raleigh, NC, ASCE.- 1975.-P. 180–230.

73 Yimsiri, S. and Soga, K. Application of micromechanics model to study anisotropy of soils at small strains. Soils and Foundations.- 2002.Vol.42.№5.-P.15–26.

74 Yimsiri, S. and Soga, K. Cross-anisotropic elastic parameters of two natural stiff clays. Geotechnique. -2011.Vol.61.№9.-P.809–814.

75 Yokota, K. and Konno, M. Dynamic Poisson's Ratio of Soil. In Proceedings of the 7th World Conference on Earthquake Engineering, pages Istanbul, Turkey, 1980.-P. 475–478.

76 Zheng, Q.-S. and Chen, T. New perspective on poisson's ratios of elastic solids. Acta Mechanica. -2001. Vol.150.№3.-P.191–195. 09.

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Анизотропты топырақтарға арналған Пуассон коэффициенттеріне қатысты кейбір бақылаулар

Аңдатпа. Топырақтың серпімділік модульдерін анықтау айқын болған көрініс. Алайда, бұл Пуассон коэффициенттеріне қатысты емес. Осыған орай, бұл жұмыста топырақ үшін Пуассон коэффициентінің мәндерін анықтауға байланысты кейбір негізгі мәселелер қарастырылады. Талқылау толықтығы үшін изотропты серпімді материалдарды идеализациялауды қамтыса да, көлденең изотропты серпімді идеализацияға баса назар аударылады.

Түйін сөздер: Пуассон коэффициенті, серпімділік, изотропия, анизотропия, ортотропия, көлденең-изотропия.

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Некоторые наблюдения относительно коэффициентов Пуассона для анизотропных грунтов

Аннотация. Определение модулей упругости грунтов в целом хорошо известно. Однако это не обязательно верно для коэффициентов Пуассона. Таким образом, в данной работе рассматриваются некоторые ключевые вопросы, связанные с определением значений коэффициента Пуассона для почв. Хотя обсуждение включает идеализации изотропных упругих материалов для полноты, акцент делается на поперечно-изотропных упругих идеализациях.

Ключевые слова: коэффициент Пуассона, упругость, изотропия, анизотропия, ортотропия, поперечная изотропия.

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