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**ТЕХНИКАЛЫҚ ҒЫЛЫМДАР ЖӘНЕ ТЕХНОЛОГИЯЛАР СЕРИЯСЫ / TECHNICAL SCIENCES AND TECHNOLOGY SERIES/ СЕРИЯ ТЕХНИЧЕСКИЕ НАУКИ И ТЕХНОЛОГИИ**

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## **Linear constraints on variables in influence diagrams for causal models**

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**Abstract.** The paper considers some ways of representing probabilistic causal models using Bayesian network theory (hereafter referred to as BN). These models describe well problems with different types of uncertainty. The theory of BN extended with some additional types of nodes called influence diagrams or IDs. Influence diagrams make it possible to consider a number of solution options, to evaluate them quantitatively, and to select the best of the considered options. However, it is practically impossible to find an optimal solution in an ID. It is not even possible to create a system of linear constraints on some variables in an ID, although there is a large class of practical problems with such constraints.

The paper describes the idea of extending ID to describe linear constraints on some variables of the BN. In the future, it will help to use the ideas of linear programming in ID to find an optimal solution in the sense of LP for problems with different types of uncertainties and causal relationships between some variables. This work has been done under grant AP19679142 "Search for optimal solutions in Bayesian networks in models with linear constraints and linear functionals. Development of algorithms and programs " (2023-2025) of MES RK. This project will develop the theory for finding optimal solutions in Bayesian networks. Optimality will be understood in the sense of linear programming - a system of linear constraints, extremum of a linear functional. The theory will be implemented in a software product.

**Keywords:** Bayesian network, directed acyclic graph, graphical model, evidence, evidence propagation, conditional probability table, influence diagrams

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### **Introduction**

Graphical models, in particular Bayesian networks and influence diagrams, can be used to solve a variety of fairly complex problems with different types of uncertainty. The basics of Bayesian networks can be found in [1, 2, 3, 4]. Such problems often involve causal relationships between different elements. These graphical models are usually based on directed acyclic graphs. The nodes in a BN are variables that are probabilistic in nature. There are different causal relationships between the variables. Calculations in BN allow us to calculate the values of some variables based on known variables, causal relationships between variables, evidence in some variables. Calculations in Bayesian networks are quite extensive. If there are 8-10 nodes in the network, manual calculations are already extremely difficult. The presence of evidence in some nodes makes calculations even more difficult. Fortunately, the theory of Bayesian network computation is well developed and implemented in many software products. In this paper we will refer to the well-known software product HUGIN EXPERT [5].

Bayesian networks are currently the subject of intense research. Interesting ideas can be seen in works [6 - 11]. Gradually, however, the capabilities of BNs became insufficient for solving many practical problems. New types of nodes were added to BNs. This made it possible to solve new classes of problems, in particular to search for and analyse solutions in problems with different causal relations and containing different types of uncertainties. It became possible to quantitatively evaluate different solutions to a problem and to select the best solution according to certain characteristics. These networks are called influence diagrams.

Then it was necessary to create various additional constraints on individual variables. Unfortunately, IDs cannot perform such operations efficiently. It is even more difficult to perform computations in constrained networks than to perform computations in conventional IDs. The issues of extending the capabilities of IDs for creating additional constraints and finding solutions to such problems are discussed in this paper.

First, the paper will construct examples to illustrate the current methods for finding the best solutions in ID. Such solutions are, of course, not generally optimal. However, in some cases the solutions may coincide with the optimal ones. In the future, the examples that are considered will be supplemented with linear constraints on some of the variables. Of course, IDs do not have mechanisms to solve such problems. However, the simplest problems can be solved artificially. As a consequence, neither the HUGIN EXPERT software package, nor any other software package that implements the ideas of IDs, allows the use of linear constraints in the problems, as well as more complex constraints.

The examples are for educational purposes only. Any questions about the suitability of the graphical model for a considered task are incorrect. The HUGIN EXPERT software is used to create the training examples.

#### **The methodology**

A tutorial example from the BN literature will be considered here. In this example, several options for solving the given influence diagram are identified and then quantitatively evaluated.

The best of the considered options is selected. However, it should be noted that the best of the considered options may not necessarily be the optimal solution for a number of reasons. For example, we may not be sure that we have considered all the options. Another reason could be that the mechanisms of influence diagrams do not adequately build the desired model. In the following, this example will be adapted in such a way that the capabilities of influence diagrams linearing, the endirple win be datpled in such a way that the expansives of initiative diagrams.  $\mathcal{L}$  methods used in this paper are Bayesian networks and their evolution  $\mathcal{L}$ ne best of the considered options is selected. However, it should be noted that the best of the  $b$  and the maximum solution will not be chosen solution will not be optimum. Another reason could be  $b$ necessary to meet in the solution to find a solution that is optimal in the solution of induction the simulatio<br>International international in such a way that the canabilities of influence diagrams

The methods used in this paper are Bayesian networks and their evolution - influence diagrams. BNs allow the user to construct a set of solutions, compare these solutions by some criterion, and select the best solution from among them. It should be noted, the chosen solution will not be optimal. In real-world problems, it is usually necessary to find a solution that is optimal in some sense. The simplest problems are linear programming problems. On the one hand, we have a set of variables that are related to each other by probabilistic relationships. On  $\,$ the other hand, a number of linear constraints on some of the variables form a set in which it is necessary to find the optimum of some linear functional. The paper considers the necessity of introducing the system of linear constraints on a set of variables into the method of influence diagrams. and the tree have a number of the cause and some of the variables form a set in which it is  $\alpha$  certain crop of a certain crop next year and some profit. However, treatment is some profit. However, treatment is some profit. However, the some profit. However, treatment is some profit. However, the some profit. H

Let us consider a well-known example from the literature on BN. The leaves of an apple tree have fallen off. The cause of the leaf drop may be either tree disease or drought, or both. The owner must decide whether to treat the tree or not. Treatment offers the hope of a certain crop next year and some profit. However, treatment is quite expensive and if treated, the risks must be assessed:  $\sigma$  ranen on. The cause of the fear urbp may be entier tree ulsease or urbught, or both. The money for cause is simply for  $\sigma$  $\sim$   $\sim$  The treatment has helped. Next year's crop will be good and will be good an

• The tree is healthy and the cause is simply insufficient watering. The money for treatment is wasted.

• The tree is sick. Treatment has helped. Next year's crop will be good and will justify the cost of treatment. tment.

• The tree is sick. The treatment has not worked. Next year's crop will be poor and the cost of the treatment has not been justified.<br>This in this influence diagram in this international contract of the state of the state of the state of the st

Figure 1 shows the influence diagram for this problem. Let us describe the variables and functions in this influence diagram.



Figure 1. Influence diagram for finding a treatment strategy for an apple tree

*Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ. Техникалық ғылымдар және технологиялар сериясы* **Figure 1. Influence diagram for finding a treatment strategy for an apple tree***ISSN: 2616-7263. eISSN: 2663-1261* 

The apple tree is observed for two years. In the first year, leaves are likely to fall (Loses) for two The apple tree is observed for two years. In the first year, leaves are fikely to fair (Loses) for two<br>reasons: the tree is sick (Sick) or the tree is withered (Dry), poorly watered. Of course, leaves may fall for a combination of these reasons. In the second year we expect a causal relationship both from<br>the old Sick node to the new Sick 1 node and from the old Dry node to the new Dry 1 node. This is the old Sick node to the new Sick\_1 node and from the old Dry node to the new Dry\_1 node. This is because, for example, if we expect a tree to be sick now, it is likely to be sick in the future.<br>Predsequence the two the comparent west the two to patrid of a possible disease. If he thinks health of the tree. We now need to specify a usefulness function that allows usefulness function that allows us

By observing the tree, the owner can treat the tree to get rid of a possible disease. If he thinks the leaf loss is due to drought, he can save his money and just wait for rain. Treating can help the tree with some probability. Treatment is quite expensive and the benefits of treatment need to dice with some probability. Treatment is quite expensive and the benefits of treatment need to be assessed. Will the crop justify the cost of treating the apple tree? The tree treatment action is now added as a decision node in the BN (Treat). This node is no longer a BN node. The Bayesian<br>Network becomes an Influence Diagram. Network becomes an Influence Diagram.

In the second year, the apple tree may be leafless for the same reasons (Disease\_1 and<br>ought 1) But the disease in the second war will denend on two reasons; whether it was sick Drought\_1). But the disease in the second year will depend on two reasons: whether it was sick in the previous year (Sick) and whether it was treated (Treat). See Figure 1.

The decision node Treat has the states "treat" and "not". We have added a link from Treat to Sick\_1. This is because we expect the treatment to affect the future health of the tree. We now need to specify a usefulness function that allows us to calculate the expected usefulness of the solution. This is done by adding auxiliary nodes to the diagram, each contributing to the overall utility. The Cost usefulness node reflects information about the cost of processing, while the Harv node represents the usefulness at harvest time. Here the usefulness depends on the health of the apple tree. To get a quantitative representation, we need to construct Conditional Probability Tables (CPTs) for all the nodes in this influence diagram.

Table 1 and Table 2 show the marginal probabilities of the variables sick and dry. Table 3 shows the probability dependence of the variable Loses on the variables Sick and Dry.

Table 1. Marginal probabilities of Sick  $T$ able 1. Marginal probabilities of Sick



Table 2. Marginal probabilities of SickDry Table 2. Marginal probabilities of SickDry



*Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ. Техникалық ғылымдар және технологиялар сериясы ISSN: 2616-7263. eISSN: 2663-1261*   $1261$ 



Table 3. Dependence of Loses on Sick and Dry Table 3. Dependence of Loses on Sick and Dry

Table 4 shows the dependence of the variable Sick\_1 on the variables Sick and Treat. Disease<br>the second vear depends on the treatment performed and whether the tree was diseased in rable 4 shows the dependence of the variable Stck\_1 on the variables Stck and Treat. Disease<br>in the second year depends on the treatment performed and whether the tree was diseased in the first year. Table 5 shows the dependence of the variable Dry\_1 on the variable Dry. Table 6 shows the dependence of the variable Dry\_1 on the variable Dry. Table 6 shows the dependence of the variable Loses\_1 on the variables Sick\_1 and Dry\_1 the depends on the treatment performed and whether the tree was diseased in the treatment performed and whether the tree was diseased in  $\epsilon$ 

Table 4. Dependence of Sick\_1 on Sick and Treat the variables Sick\_1 and Dry\_1  $T$ Treat treat not Sick sick not sick not  $0.2$ sick  $0.01$ 0.99  $0.02$ not  $0.8$ 0.99  $0.01$ 0.98

Table 5. Dependence of Dry\_1 on the variable Dry Table 5. Dependence of Dry\_1 on the variable Dry Table 5. Dependence of Dry\_1 on the variable Dry



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Table 6. Dependence of Loses\_1 on variables Sick\_1 and Dry\_1

Table 7. Willingness to pay for tree treatment Table 7. Willingness to pay for tree treatment Table 7. Willingness to pay for tree treatment



Table 7 describes the owner's marginal willingness to pay for treatment of the tree. Table 8 shows table 7 describes the owner s marginal winnighess to pay for treatment or the tree. Table 6 shows<br>that it would cost \$8,000 to treat an apple tree. Table 9 shows the profit from a sick and healthy tree. Table 8. Treating an apple tree will cost \$8,000 dollars  $T$ able 7 describes the owner's marginal will be owner to pay for the owner's marginal will be the owner of the owner treatment o to treat an apple tree. Table 9 shows the profit from a sick and healthy tree.<br>pple tree will cost \$8,000 dollars



pay for tree treatment



Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ. №2(147)/ 2024 193<br>Техникалык ғылымдар және технологиялар сериясы Техникалық ғылымдар және технологиялар сериясы<br>ISSN: 2616 7262 <u>eJSSN: 2662 1261</u> *ISSN: 2616-7263. eISSN: 2663-1261*   $\alpha$ погиялар сериясы

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Let's evaluate several strategies for the behaviour of the apple tree owner.

1. Strategy 1: The owner does not spend money on treating the apple tree, believing that the cause of the leaf fall is drought. The solution is shown in Figure 2. Figure 2.



Figure 2. The owner does not spend money on treatment **Figure 2. The owner does not spend money on treatment**

The expected profit in this case is 18011 dollars. The expected profit in this case is 18011 dollars.

2. Strategy 2. The owner spends no money on treating the apple tree, believing that the leaves 2. Strategy 2. The owner spends no money on treating the apple tree, believing that are falling because of the drought. But it turns out that the tree is sick. The solution is shown in Figure 3.  $T_{\rm eff}$  solution is shown in Figure 3.



Figure 3. The owner does not spend money on treatment, but the apple tree is sick **Figure 3. The owner does not spend money on treatment, but the apple tree is** 

The expected profit in this case is \$3,000. A sick tree will yield little profit. The expected profit in this case is 3,000. A since the expected profit in this case is 3,000. A sick tree will



3. Strategy 3. The owner treats the apple tree, and the tree is sick. 3. Strategy 3. The owner treats the apple tree, and the tree is sick.

Figure 4. The owner treats the apple tree, but the tree is not cured **Figure 4. The owner treats the apple tree, but the tree is not cured**

The expected profit in this case is -5000 dollars. The diseased tree will yield little profit<br>  $\sum_{n=1}^{\infty}$ (\$3000), and \$8000 was spent on treatment. The solution is shown in Figure 4.

4. Strategy 4. The owner treats an apple tree, the tree was sick, but was cured. The expected 4. Strategy 4. The owner treats an apple tree, the tree was sick, but was cured. The expected profit is \$12000 (20000 from the sale - 8000 for the cure). The solution is shown in Figure 5. is shown in Figure 5.



Figure 5. An owner treats a sick apple tree, the tree has been cured **Figure 5. An owner treats a sick apple tree, the tree has been cured**

Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ. №**2(147)/ 2024 195** Техникалық ғылымдар және технологиялар сериясы<br>ISSN· 2616-7263 eISSN· 2663-1261 *ISSN: 2616-7263. eISSN: 2663-1261* 

The examples that have been considered show quite clearly the scope of influence diagrams in solving practical problems. However, many questions arise about the scope of influence in solving practical problems. However, many questions arise about the scope of influence diagrams. Let us consider just a few of these questions. different costs and different treatment  $\alpha$  and different treatment effects.

igrams. Let us consider just a lew of these questions.<br>Is it possible to specify the cost function for tree treatment in detail? Within reasonable limits it is. For example, there are several ways to treat a tree, with different costs and different<br>treatment efficiencies. For example: treatment efficiencies. For example:

o Do not treat the tree, no treatment cost.

o Spend at least \$3000 with a probability of curing the tree of about 0.6.

o Spend \$5000 with a probability of curing the tree of about 0.8. with a probability of curing the tree of about 0.8.<br>with a probability of curing the tree greater than 0.9.

o Spend \$8000 with a probability of curing the tree greater than 0.9.

The Treat node and its associated Cost and Sick\_1 nodes will change. The conditional probability tables (10, 11, 12) for these changes are given below.



Table 10. Willingness to pay for tree treatment **Table 10. Willingness to pay for tree treatment**

#### Table 11. Cost of treatment of apple tree



# Table 12. Probabilities of apple tree recovery **Table 12. Probabilities of apple tree recovery**



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*Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ.* The user only selects the best option from the options considered. There is no *Техникалық ғылымдар және технологиялар сериясы ISSN: 2616-7263. eISSN: 2663-1261* 

The user only selects the best option from the options considered. There is no certainty that the most interesting options will be omitted. It would be nice to have a mechanism to point the user to more interesting options. Unfortunately, IDs do not have such capabilities.

In practical problems, there are often linear constraints on some (not all) variables in a ID. For example, a owner may have several trees that are specific and require different costs. However, resources are finite. The natural question of the most efficient allocation of resources cannot be solved by a ID. In the simplest cases, such problems can be solved using artificial methods. However, there is no general approach to solving such problems in IDs. Figure 6 shows an example of such a problem. The solving such a solving such a shows an example of such a problem.



Figure 6. Example with two diseased trees **Figure 6. Example with two diseased trees**

The owner has two diseased trees: an apple and a pear. The owner can start treating these trees. However, there is no money to fully treat both trees - only \$8000 is available. You can either refuse to treat the trees or choose one of three treatment modes for each tree. Each<br>treatment mode costs a certain amount of money. The less a treatment costs, the less likely it treatment mode costs a certain amount of money. The less a treatment costs, the less likely it is that the tree will be cured. Figure 6 shows a Bayesian network with two diseased trees. We leave the basic probabilistic constraints for this example the same as in the example discussed eart the basic probabilistic constraints for this example the same as in the example diseassed<br>earlier. Only new relationships between the variables are given below.



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*Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ.* treatment options for pear tree. Table 15 shows the cost of treating the apple tree *The typicality amolities of typicall channon, yndeepeamolinii, m.e.m. m.e.com* amolitic treatment of treatment o<br>Техникалық ғылымдар және технологиялар сериясы *ISSN: 2616-7263. eISSN: 2663-1261*   $\frac{2003 - 1201}{\pi}$ 

Table 13 summarizes the treatment options for apple trees. Table 14 shows the treatment options for pear tree. Table 15 shows the cost of treating the apple tree depending on the<br>tractrocate with a Table 16 shows the seat of treating the near tree depending on the treatment. treatment option. Table 16 shows the cost of treating the pear tree depending on the treatment treatment option. Table 16 shows the cost of treating the pear tree depending on the treatment<br>option and the funds already spent on treating the apple tree. Since the treatment funds are only \$8000, the excess over this amount is represented by an arbitrary, rather large number. In our case, the loss is \$99,99999. Figure 7 shows the solution to this problem if the tree owner our case, the loss is \$99,99999. Figure 7 shows the solution to this problem if the tree owner<br>refuses any treatment, hoping that the leaves will fall only from lack of irrigation. The expected refuses any treatment, hoping that the reaves win fail only from fack of fix agation. The expected gain is  $18011 + 10080 = 28091$  dollars.

Table 14. Pear tree treatment options **represents in our case, the loss is the loss is the loss is the loss is**  $\alpha$ **Table 14. Pear tree treatment options Table 14. Pear tree treatment options**

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Table 15. Cost of treatment of an apple tree  **Table 15. Cost of treatment of an apple tree Table 15. Cost of treatment of an apple tree**



Table 16. Cost of treatment of pear tree  **Table 16. Cost of treatment of pear tree Table 16. Cost of treatment of pear tree Table 16. Cost of treatment of pear tree**





Figure 7. Variant of solving the problem with two diseased trees

## **Findings/Discussion Findings/Discussion**

Two examples of the use of IDs are considered in this paper. These examples illustrate the range of problems that can be solved by IDs methods. The second example considers the range of problems that can be solved by IDs methods. The second example considers the are range of problems that can be sorroa by the memeter rife eccent entingly constants the simplest variant of a linear constraint on two variables. An artificial way of solving this problem using IDs is considered. It is concluded that it is impossible to use the IDs mechanism to solve problems with linear constraints on some variables. Two examples of the use of IDs are considered in this paper. These examples

#### **Conclusion** some variables. The development of ideas of ideas of ideas of ideas of constructing linear constraints for some variables of  $\alpha$

The development of ideas of constructing linear constraints for some variables of influence diagrams is the first step towards solving practical problems of finding the optimal solution of linear programming problems in the presence of various types of uncertainties. It is also assumed that some variables may have probabilistic relationships among themselves.<br>.

## **Acknowledgement, conflict of interests Acknowledgement, conflict of interests**

The paper considers the possibilities of using the mechanism of ID to solve problems where some variables are bound by linear constraints. These problems will be solved within the grant project AP19679142 "Search for optimal solutions in Bayesian networks in models with linear project in the constraints in models in models with linear constraints in models with linear functionals. Development of algorithms and programs " (2023-2025) of MES RK. The paper considers the possibilities of using the mechanism of ID to solve problems The paper considers are possibilities or asing the incentius in the to solve problems wilcre  $R$ K.

## **Author's contribution**

**А. Shayakhmetova, A. Akhmetova –** general guidance, problem setting, solving theoretical issues, calculations, conclusion.

**A. Abdildayeva, N. Litvinenko, A. Zakirova –** formula checking, calculation checking, article design, typing.

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**Аңдатпа.** Мақалада Байес желілерінің теориясын (бұдан әрі БЖ) қолдана отырып, ықтималдық себеп-салдарлық модельдерді ұсынудың кейбір әдістері қарастырылады. Бұл модельдер әртүрлі белгісіздіктері бар тапсырмаларды жақсы сипаттайды. БЖ теориясы түйіндердің кейбір қосымша түрлерімен толықтырылған әсер ету диаграммалары немесе ӘД деп аталады. ӘД көптеген шешімдерді қарастыруға, оларды сандық бағалауға және қарастырылған нұсқалардың ішінен ең жақсысын таңдауға мүмкіндік береді. Дегенмен, ӘД-де оңтайлы шешімді табу мүмкін

емес. ӘД-де кейбір айнымалыларға сызықтық шектеулер жүйесін құру мүмкін емес, дегенмен мұндай шектеулермен практикалық есептердің үлкен класы бар.

Мақалада кейбір БЖ айнымалыларындағы сызықтық шектеулерді сипаттау үшін ӘД-ні кеңейту идеясы сипатталған. Болашақта бұл әр түрлі белгісіздіктер мен кейбір айнымалылар арасындағы себеп-салдарлық байланыстары бар есептер үшін сызықтық бағдарламалау мағынасында оңтайлы шешімді табу үшін ӘД-да сызықтық бағдарламалау идеяларын қолдануға көмектеседі. Жұмыс «AP19679142 Сызықтық шектеулер мен сызықтық функционалдығы бар модельдерде Байес желілерінде оңтайлы шешімдерді табу. Алгоритмдер мен бағдарламаларды құру» гранттық қаржыландыру негізінде орындалды (2023-2025жж.). Оңтайлылық сызықтық бағдарламалау мағынасында -сызықтық шектеулер жүйесі, сызықтық функционалдың экстремумы түсініледі. Теория бағдарламалық өнімде жүзеге асырылады.

**Кілттік сөздер:** Байес желісі, бағытталған ациклді график, графикалық модель, дәлел, дәлелдемелерді тарату, шартты Ықтималдықтар кестесі, әсер ету диаграммалары.

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#### **Линейные ограничения переменных в диаграммах влияния для причинно-следственных моделей**

**Аннотация.** В статье рассматриваются некоторые способы представления вероятностных причинно-следственных моделей с использованием теории байесовских сетей (в дальнейшем БС). Данные модели хорошо описывают задачи с различными видами неопределенностей. Теория БС, дополненная некоторыми дополнительными типами узлов, называется диаграммами влияния, или ДВ. ДВ позволяют рассмотреть некоторое множество вариантов решений, оценить их количественно и выбрать лучшее из рассмотренных вариантов. Однако поиск оптимального решения в ДВ практически невозможен. В ДВ невозможно даже создать систему линейных ограничений на некоторые переменные, хотя существует большой класс практических задач с такими ограничениями. В статье описывается идея расширения ДВ для описания линейных ограничений на некоторые переменные БС. В дальнейшем это поможет использовать идеи линейного программирования в ДВ для поиска оптимального решения в смысле ЛП для задач с различными видами неопределенностей и причинно-следственными связями между некоторыми переменными. Работа написана в рамках грантового финансирования AP19679142 «Поиск оптимальных решений в байесовских сетях в моделях с линейными ограничениями и линейными функционалами. Разработка алгоритмов и программ» (2023-2025гг.) МОНВ РК. В рамках данного проекта будет разработана теория, позволяющая находить оптимальные решения в байесовских сетях. Оптимальность будет пониматься в смысле линейного программирования – система линейных ограничений, экстремум линейного функционала. Теория будет реализована в программном продукте.

**Ключевые слова:** Байесовская сеть, направленный ациклический граф, графическая модель, свидетельство, распространение свидетельств, таблица условных вероятностей, диаграммы влияния.

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