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Article

## Mathematical and computer modeling of transmission with non-traditional engagement for mining equipment drive

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**Abstract.** In modern gear mechanics, transmission mechanisms play a crucial role in converting the rotational motion of a driving shaft into the rotational motion of another shaft with varying angular speeds and torque. To achieve optimal designs for the next generation of transmission mechanisms, it is essential to develop mathematical models of their dynamic behavior, conduct computer simulations of the meshing geometry of key components, and visualize the operation of the mechanism. Despite the widespread use of involute gearing in mechanical transmissions, there is ongoing research into new types of gearing that offer advantages over traditional systems. The main challenges facing the industry include increasing the gear ratio in a single stage, enhancing load capacity, and improving efficiency compared to standard gear transmissions. This paper presents the results of mathematical and computer modeling, along with a comparative analysis of the eccentric-cycloid (EC) engagement with the involute gear transmission. Through analytical calculations, the energy-force parameters of the EC gearbox were determined, equivalent stresses and static deflections of transmission shafts were obtained. The paper includes the results of static analysis of elements of the new EC transmission, as well as an algorithm for computer modeling of contact stresses occurring in the engagement. Conclusively, by comparing contact stresses in traditional involute gear transmission, calculated using various analytical methods, with those in EC engagement determined through computer simulation, the advantages of the new transmission type and its potential application in mining equipment transmission mechanisms are highlighted.

**Keywords:** eccentric-cycloid transmissions, involute engagement, Novikov gearing, contact stresses, centrifugal forces.

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## Introduction

In modern mechanical engineering, transmission mechanisms play a pivotal role in converting the rotational motion of a driving shaft into rotational motion of another shaft with varying angular velocities and torque. To achieve optimal designs for transmission mechanisms of the new generation, it is necessary to create mathematical models of their dynamic behavior, perform computer modeling of the engagement geometry of key components, and visualize the mechanism's operation process. Concurrently with the development and improvement of the widely used involute gear engagement in mechanical transmissions, there is a continuous search for new types of engagement possessing various advantages over the involute gear system.

Modern industry faces several key challenges, including increasing the gear ratio in a single stage, enhancing load capacity, and efficiency compared to standard toothed transmissions. The standard involute profile, while widely used, has geometric limitations for external toothed wheels, including undercuts and a small radius of curvature near the base circle. Special toothed wheels with adapted profile geometry are employed to overcome these limitations, opening new possibilities in design. Technological innovations such as additive manufacturing and 5-axis milling enable cost-effective production of such special toothed wheels. Unlike involute toothed wheels, the geometric description of non-involute toothed wheels is often non-standardized, making it difficult to determine their properties. One such special tooth profile is the eccentric cycloidal gear (EC-gear), which offers advantages over standard involute gears in certain applications. This study presents a geometric description of the EC-toothed transmission based on a defined set of parameters, including parameters describing the characteristics of the gear transmission. This parametric description allows for the analytical determination of contact geometry and characteristics without load, which is useful for creating gear transmissions that meet practical needs [1-5].

When designing any gear transmissions, especially non-traditional ones, one of the important parameters is the determination of contact stresses and contact fatigue strength of the gear teeth.

In the article [6] a mathematical model of the operation of a reducer using a new type of engagement of working wheels, one of which is a helical eccentric, and the profile of the other is based on a cycloidal curve, is built. Such engagement has increased force characteristics and allows for high gear ratios in a single stage. A computer program illustrating the kinematically coherent motion of ideal geometric figures - end sections of the working mechanism, is created, allowing for the determination of numerical characteristics necessary for design, but the article lacks research results on testing the new transmission for contact strength.

The stress-strain state (SSS) of the contact engagement largely determines such an important parameter of the reducer as the efficiency coefficient. Analytical calculation of contact loads always poses significant difficulties for both involute and other engagements with a large contact area and a share of transmitted torque during sliding friction. The emergence of computer programs has enabled modeling the SSS of contact, explaining the physical essence of the engagement efficiency, and using the obtained results for verification calculations.

The objective of this article is to conduct an analysis of the static strength of elements of the new EC engagement and to determine the level of contact stresses arising in the new type of

transmission, as well as to perform a comparative analysis of existing studies on determining contact stresses.

### **The methodology**

New EC gear reducers have established themselves as precise and rigid mechanisms, combining a good ratio of transmitted torque, overall dimensions, and weight. The main advantage is the wide range of gear ratios. Gear reducers are most commonly used in modern developing industries: CNC machines, automatic lines, transport machinery, and robotics. The efficiency coefficient of such a transmission reaches 90%.

Due to the reduction in the number of stages in the EC gear reducer, fewer bearings are used, which increases its efficiency. The developed gear reducer has a service life one and a half times longer. The driven wheel of the EC has a profile enveloping a family of circles in different phases of engagement and represents a cycloidal curve, which is an equidistant. Figure 1 shows the diagram of the EC transmission.

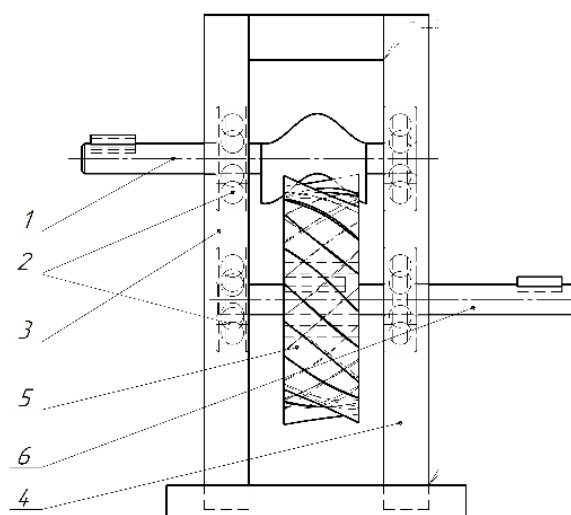


Figure 1. EC transmission: 1 – driving shaft-eccentric, 2 – bearings, 3 – left stand, 4 – right stand, 5 – driven wheel, 6 – output shaft

The profile of the driving wheel in the end section is a circle eccentrically displaced from the axis of rotation of the wheel by a distance  $\varepsilon$ . The curvilinear helical profile of the wheel is formed by sequentially and continuously displacing this circle along the axis of the wheel with simultaneous rotation around the same axis.

### **Geometric Model of the Mechanism**

The geometric model of the EC mechanism is shown in Figure 2. The tooth profile of the smaller wheel 1 in the end section is a circle  $D$  with a diameter  $d=2r$ , eccentrically displaced by a distance  $\varepsilon$  relative to the axis of rotation of the wheel  $OO_1$ . The curved profile of wheel 1 is

formed by the sequential and continuous displacement of this circle along the axis of wheel  $OO_1$  with simultaneous rotation around this same axis. Thus, the tooth surface of wheel 1 forms a helical eccentric P.

The tooth profile of the larger wheel 2 in the end section is conjugated with the eccentrically displaced circle D of wheel 1. The profile is constructed as the envelope of a family of eccentric circles in different phases of engagement and represents a cycloidal curve G, which is an equidistant of the epicycloid [3]. The helical curved surface of the teeth of wheel 2 is formed similarly to the tooth surface of wheel 1 by the sequential and continuous rotation of the cycloidal end sections of the wheel around the axis  $CC_1$  of wheel 2. The helical surfaces of wheels 1 and 2 have opposite directions of rotation.

The general view of the reducer with plane P perpendicular to the axes of the wheels is shown in Fig. 2, and a fragment of the contact area of the worm element with the larger wheel is shown in Fig. 2. The toothed profile of the smaller wheel 1 in the end section is a circle D of diameter  $d=2r$ , eccentrically shifted by a distance  $\varepsilon$  relative to the axis of rotation of the wheel  $OO_1$ . The curvilinear profile of the wheel 1 is formed by sequentially and continuously displacing this circle along the axis of the wheel  $OO_1$  with simultaneous rotation around the same axis. Thus, the tooth surface of wheel 1 forms a helical eccentric P.

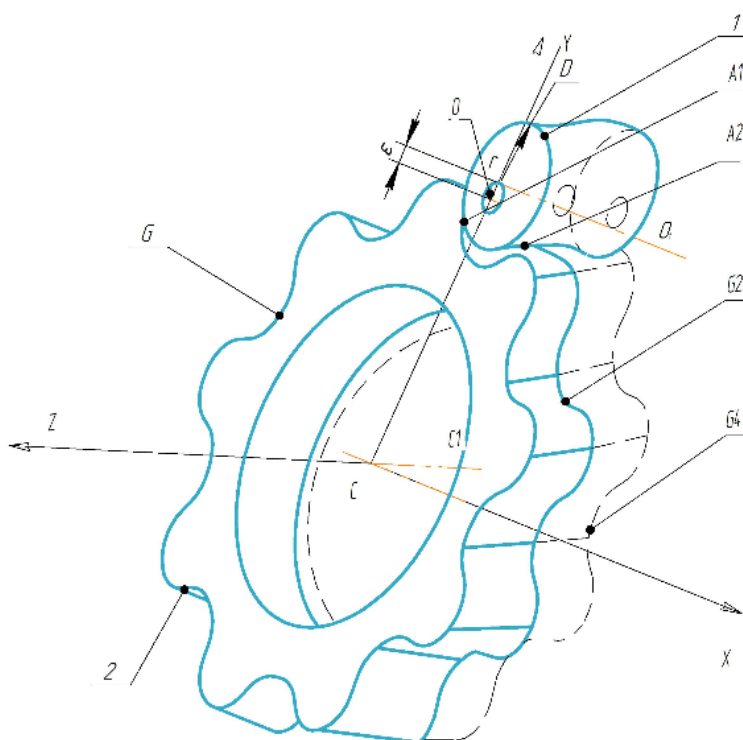


Figure 2. Diagram of the eccentric cycloid meshing

### Determination of EC Transmission Parameters

The input parameters for determining the geometric dimensions of the EC gearing of the

reducer for the gantry crane movement mechanism were taken according to the technical specifications:

Gear ratio  $i = 9$

Torque  $T_{\max} = 2500 \text{ N}\cdot\text{m}$

Based on the fundamental equations of the epicycloid (1), (2), the parameters of the driving and driven wheels were determined.

$$X = (45 + 5) \cdot \cos \pi \cdot u + 2 \cdot \cos \left( (45 + 5) \cdot 2 \cdot \pi \cdot \frac{u}{5} \right) \quad (1)$$

$$Y = (45 + 5) \cdot \sin \pi \cdot u + 2 \cdot \sin \left( (45 + 5) \cdot 2 \cdot \pi \cdot \frac{u}{5} \right) \quad (2)$$

Wheel diameter  $R = 150 \text{ mm}$

Number of teeth of the larger wheel  $n = 16$

Profile diameter of the smaller wheel  $r = 40 \text{ mm}$

Eccentricity of the eccentric shaft  $e = 5 \text{ mm}$

Based on the calculated geometric parameters, engineering documentation for the EC reducer was created (Figure 3).

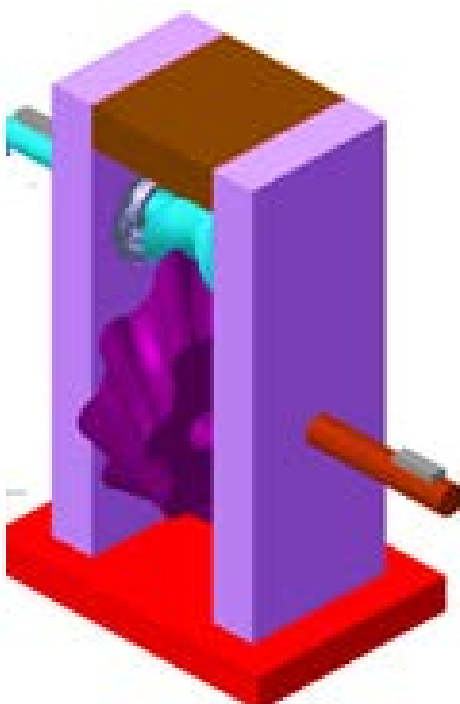


Figure 3. 3D model of the EC reducer

The designed reducer is a single-stage cylindrical reducer with a mass of 41 kg. The reducer is intended to replace a cylindrical two-stage involute reducer, which has a mass of 96 kg.

Mathematical model of contact in the new evolvent-cycloid transmission.

As seen from the construction scheme (Figure 2) of the tooth surfaces of wheels 1 and 2, the tooth profile of wheel 1 in any end section is represented by the eccentrically displaced circle D, while the profile of wheel 2 is represented by the rotated cycloidal curve G. In any end section, circle D has a contact point A with the corresponding cycloidal curve. The helical tooth of wheel 1 simultaneously has multiple contact points with the helical cycloidal tooth of wheel 2. These points form a continuous helical (with variable curvature) contact line  $AA_2A$ .

The coordinates of the contact point A of circle D with the cycloidal curve G are found as the sum of the radius vector of the center of circle D and the vector directed along the normal to this circle at the contact point, having a length equal to the radius of circle D. To find this normal, it is not necessary to resort to differentiation – it is sufficient to apply the property of cycloidal curves: the normal at an arbitrary point of such a curve passes through the pole (the point of tangency of the rolling circles used to generate the initial cycloidal curve). The line  $AA_2A_4$  is constructed using the interpolation function of the contact point array for adjacent end sections, built into the MathCad package. The resulting vector function  $K\vartheta(\vartheta)$  ( $\vartheta=0, \dots, 2\pi$  – the angle of rotation of circle D around the axis  $OO_1$ , which produces the corresponding end section) of the points on line  $AA_2A_4$  allows differentiation using the symbolic processor of the MathCad package to determine the curvature at each point of this line at any given time. This curvature turns out to be variable, i.e., the contact line is not helical.

According to Hertz's theory, the deformation state of contact between two cylindrical bodies of radii  $R_1$  and  $R_2$  arising from normal forces P, is shown in Figure 2[7-8].

When both points  $A_1$  and  $A_2$  of the bodies shift toward point O along the z – axis by distances  $\delta\delta_1$  and  $\delta\delta_2$  respectively, the average contact pressure  $p_m$  is determined by the equation:

$$p_m = a \left( \frac{1}{R_1} + \frac{1}{R_2} \right) / \left( \frac{1}{E_1} + \frac{1}{E_2} \right) \quad (3)$$

Thus, the contact pressure and the stresses it induces increase proportionally to the linear size of the contact area. For cylinder contacts, the load per unit length of the axis is:

$$P = 2ap_m \quad (4)$$

Then, from equation (5), we derive:

$$p_m = \left[ P \left( \frac{1}{R_1} + \frac{1}{R_2} \right) / \left( \frac{1}{E_1} + \frac{1}{E_2} \right) \right]^{\frac{1}{2}} \quad (5)$$

The dimensions of the semi-axes of the contact ellipse can be determined by the expressions:

$$a = m_3 \sqrt{\frac{3\pi P(k_1 + k_2)}{4(A + B)}}; b = n_3 \sqrt{\frac{3\pi P(k_1 + k_2)}{4(A + B)}} \quad (6)$$

Here, m and n are coefficients dependent on  $\cos \varphi = \frac{B - A}{B + A}$ , determined by the expressions:

$$m = \sqrt[3]{\frac{2}{\pi} D(e, \varphi) \frac{A + B}{B}}; n = \sqrt{\frac{2}{\pi} [E(e, \varphi) - D(e, \varphi)] \sqrt{1 - e^2} \frac{A + B}{B}} \quad (7)$$

Where A and B are constant coefficients dependent on the magnitudes of the principal curvatures of the contacting bodies and the angle between the planes of the principal curvatures of their surfaces, and  $E(e, \varphi)$  and  $D(e, \varphi)$  are elliptic D integrals given by:

$$E(e, \varphi) = \int_0^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 \varphi} d\varphi \quad (8)$$

$$D(e, \varphi) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - e^2 \sin^2 \varphi}} \quad (9)$$

Here,  $R_1$  and  $R_2$  are the radii of the cylinder,  $k_1$  and  $k_2$  are constants determined by the equations:

$$k_1 = \frac{1 - \mathcal{G}_1^2}{\pi E_1}, k_2 = \frac{1 - \mathcal{G}_2^2}{\pi E_2} \quad (10)$$

From equations (5) – (6), it follows that the width of the contact ellipse and the contact pressure increase as the square root of the applied load [10-13].

Hertz's theory defines the regularities of contact stresses and deformations as the compressive load increases and determines the influence of the surface curvatures and the elasticity moduli of the contacting bodies [7].

To find the contact stresses at the points along the line, it is necessary to know the radius of curvature of the line on the larger tooth 2, which is obtained by the end section corresponding to the contact point, i.e., at a given angle  $\nu$ . This line is the result of rotating the initial line G by an angle:

$$\frac{-(\mathcal{G} + \delta)}{z_2}, \quad (11)$$

where  $\delta$  is the rotation angle of the generator. The radii of curvature are calculated by the usual formula:

$$R(\vartheta, \delta) = \frac{(X'(\varphi(\vartheta, \delta))^2 + Y'(\varphi(\vartheta, \delta))^2)^{\frac{3}{2}}}{X'(\varphi(\vartheta, \delta))Y''(\varphi(\vartheta, \delta)) - X''(\varphi(\vartheta, \delta))Y'(\varphi(\vartheta, \delta))}, \quad (12)$$

where:

$$\varphi(\vartheta, \delta) = \frac{z_2 + 1}{z_2}(\vartheta + \delta), \quad (13)$$

and  $X(\varphi(v, \delta)), Y(\varphi(v, \delta))$  – are the coordinates of the contact point on the corresponding equidistant.

The formula for calculating the forces at the contact points at the rotor rotation angle  $\delta$  takes the integral form:

$$F(\vartheta, \delta) = \frac{M \sin(\gamma(\vartheta, \delta))}{\int_{\delta}^{\delta+\pi} \sqrt{X(\varphi(\vartheta, \delta)) - a)^2 + Y(\varphi(\vartheta, \delta)) - a)^2 \sin^2(\gamma(\vartheta, \delta))} d\vartheta, \quad (14)$$

where  $M$  is the input torque on the generator, and  $\gamma(v, \delta)$  is the angle between the radius vector of the contact point.

The loading scheme of the teeth in the EC gearing is schematically shown in Figure 4.

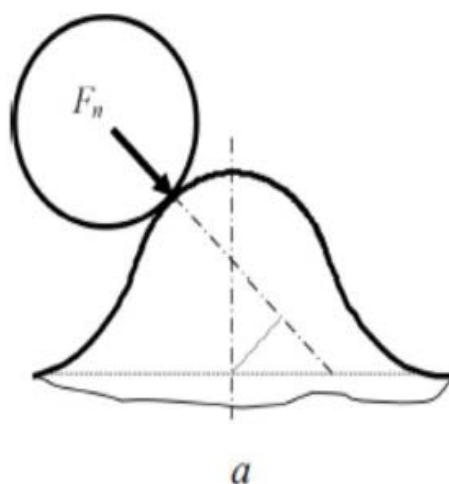


Figure 4. Tooth loading schemes



As a result of analytical calculation, the following energy-force parameters of the EC reducer were determined:

1 Output shaft rotation speed:  $n=725 \text{ rpm}$

2 Torque on the input shaft of the reducer:  $T=4.8 \text{ kNm}$

3 Sum of the projections of all forces in engagement of the eccentric and driven wheel teeth on the x-axis:

$$\sum F_{ix} = 4,025 \text{ kN}$$

4 Sum of the projections of forces in engagement of the teeth on the y-axis:

$$\sum F_{iy} = -2,850.5 \text{ kN}$$

5 Electric motor power:  $N=24 \text{ kW}$

### **Computer Modeling of Contact in EC Engagement**

Thanks to the application of computer technology, the possibilities for calculating contact stresses in mechanisms have significantly expanded. The NASTRAN/MARC program supports three contact models: node-to-node, node-to-surface, and surface-to-surface. Each type of model uses different types of contact elements. The finite element model recognizes the contact pair by the presence of contact elements that are applied to those parts of the model that will be analyzed for interaction. To form a contact pair, these elements use the concepts of "target surface" and "contact surface". For determining two-dimensional contact pairs, finite elements CONTA and TARGE are used, and for three-dimensional contact pairs, CONTA174 and TARGE170 are used.

The main steps for performing surface-to-surface contact analysis are outlined below:

Creating a geometric model and meshing;

Designing the contact and target surfaces;

Defining the contact and target surfaces;

Setting real constants;

Setting the necessary boundary conditions and solution options;

Solving the contact problem;

Analyzing the results. Boundary conditions for static analysis and calculation of contact stresses are summarized in Table 1.

Table 1. Boundary conditions

№	Parameter	Value
1	Material	Steel
2	Elastic modulus E	$2.10^{11} \text{ Pa}$
3	Poisson's ratio	0.3
4	Support 1	Hinge with rotation around the model axis
5	Axial force	4.025kN
6	Radial force	2850.5kN

7	Calculation type	Static
8	Output shaft rotation speed	$n=725 \text{ rpm}$

Using this algorithm, a methodology for solving contact problems based on the finite element method in the NASTRAN/MARC software was developed [9]. This methodology was tested on contact problems, the solutions of which were obtained by classical mechanics methods.

## Findings/Discussion

### Analysis of Computer Modeling Results

Choosing the FEA system NASTRAN/PATRAN for strength analysis, we present the schematic diagram of the rotor (Figure 3) as a finite element model. The schematic diagram represents the elastic-mass characteristics of the mechanical system "rotor-support." There are several approaches to creating FEA models [14]. Initially, we create the geometry of the rotor according to the working drawings (Figure 5).

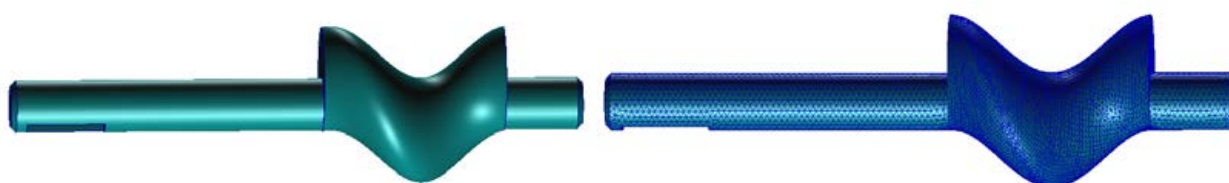


Figure 5. 3D model and finite element mesh of the components of the eccentric (EC) reducer

Equivalent stresses and static deflection values of the shaft were obtained as a result of the static strength calculation.

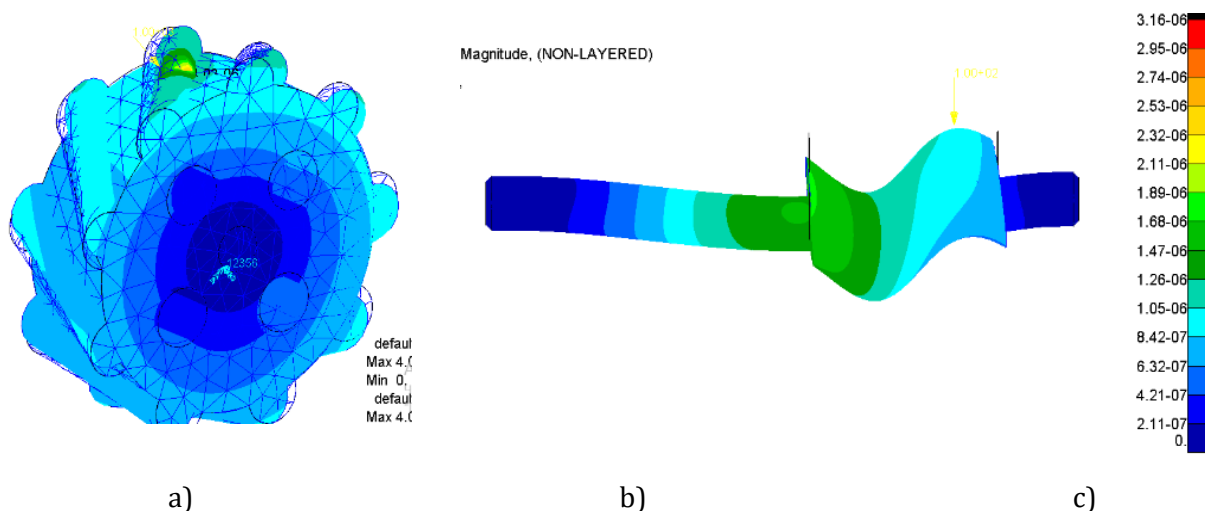


Figure 6. Stress and displacement diagrams in the driven wheel of the EC reducer

The analysis of the results revealed a sufficient safety factor for the driven wheel. According to the diagram (Figure 6a), the maximum stress  $\sigma_{\max} = 195 \text{ MPa}$  and the static deflection  $\delta_{cm} = 9.29 \cdot 10^{-5} \text{ m}$ . The allowable stress for steel is  $[\sigma_-] = 450 \text{ MPa}$ , resulting in a safety factor of 2.3.

The stress analysis diagrams of the main shaft (Figure 6b) indicate a maximum stress of  $\sigma_{\max} = 44 \text{ MPa}$  and a displacement of  $\delta_{cm} = 3.16 \cdot 10^{-6} \text{ m}$ , yielding a safety factor of 3.2.

The stress analysis diagrams of the reducer frame show a sufficient safety factor. According to the diagram (Figure 6c), the maximum stress and the static deflection  $\delta_{cm} = 2.43 \cdot 10^{-6} \text{ m}$ . The allowable stress for steel is  $[\sigma_-] = 450 \text{ MPa}$ , resulting in a safety factor of 8.3.

### Determination of Contact Stresses in Meshing in the NASTRAN/MARC System

Eccentric (EC) transmissions experience significant axial loads, necessitating appropriate design of bearing assemblies. These loads lead to high contact pressures and significant friction losses, resulting in low efficiency.

Contact pressure modeling was performed using the high-level CAD system NASTRAN/MARC, a nonlinear solver for determining contact parameters between two bodies[7-8]. The results in the form of diagrams are presented in Figure 7.

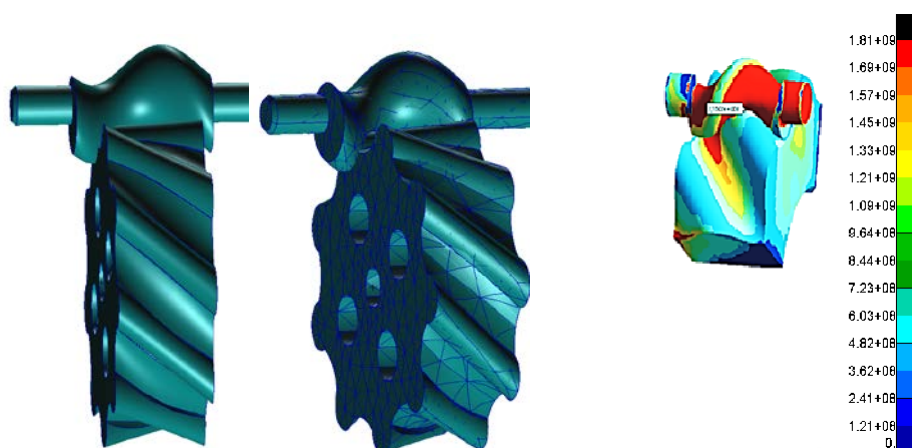


Figure 7. Contact stress diagram

The maximum pressure in the EC contact is 1810 MPa, with the greatest tensile stress on the EC wheel being 1342 MPa, and on the EC eccentric 714 MPa. The EC eccentric requires surface hardening such as cementation or SHKh15 steel with volumetric strengthening. The complex configuration of the eccentric creates a triaxial stress state, which, combined with cyclic loading, reduces its service life.

A comparative review of mathematical models of reducers with non-traditional gearing was of interest. The inadequacy of the theory in its classical form is confirmed by calculated values of  $p_{\max}$ , found using various authors' methods. Comparison of transmissions based on key parameters is presented in Table 2.

Table 2. Comparison of transmissions by basic parameters

Measured Parameters	Information Sources				
	Herz	Kovalyov M.N	Makushin M.I.	VNIINMASH	IMASH
$p_{\max}, \text{MPa}$	9771	5843	9818	4473	5706
$\varphi_{\sigma} = \frac{\sigma_{\max}}{\sigma_{\max}^*}$	14,00	5,50	14,06	6,41	4,60
$\varphi_F = \varphi_{\sigma}^3$	2743	167	2783	263	97

From Table 2, it follows that calculated stress values are approximately three times higher than the stress observed in EC transmissions. This increase in  $\varphi_{\sigma}$  corresponds to a decrease in the load capacity of the EC mesh due to contact stresses.

### Conclusion

Based on the analysis of static strength and contact stresses, which are the main causes of failure in reducers with involute gearings, a mathematical and computer model of a reducer with involute-cycloidal gearings has been proposed. Gearings with involute-cycloidal tooth profiles, widely used, are expected to become one of the most effective and reliable types of gearings used in various mechanical systems. The new type of EC gearing has a high gear ratio with minimal dimensions. The teeth have a large effective radius of curvature, increasing contact strength, while the tooth shape ensures high bending strength. In our opinion, EC gearing can become a serious competitor not only to traditional involute gearing but also to other types of gearings currently being developed. They provide high torque transmission accuracy, minimal noise and vibration levels, and the ability to transmit large loads in compact sizes.

The material presented in our publication can be used in the design process of reducers, transmissions, and other transmission mechanisms used in mining equipment, as a primary or additional source of reference information.

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### The contribution of the authors:

**Isametova M.E.** – the idea author who proposed the direction of the work.

**Seiitkazy N.S.** – contributed to the methodology of the article and completed the work.

**Saidinbayeva N.D.** – gathered data and contributed to editing the work.

**Abilezova G.S.** – contributed to visualization and analysis tasks.

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### **Тау-кен техникасының жетегіне арналған дәстүрлі емес ілінісі бар берілісті математикалық және компьютерлік модельдеу**

**Аңдатпа.** Заманауи беріліс механикасында беріліс механизмдері жетекші біліктің айналу қозғалысын бұрыштық жылдамдықтары мен моменті өзгертін басқа біліктің айналу қозғалысына түрлендіруде шешуші рөл атқарады. Жаңа буын беріліс механизмдерінің оңтайлы конструкцияларына қол жеткізу үшін олардың динамикалық өзгерістерінің математикалық үлгілерін құру, негізгі компоненттердің қосылу геометриясының компьютерлік модельдеуін жүргізу және механизмнің жұмыс процесін визуализациялау қажет. Механикалық беріліс қорабында эвольвентті берілістің кеңінен қолданылуына қарамастан, дәстүрлі жүйеден артықшылығы бар берілістердің жаңа түрлерін іздеу жалғасуда. Өнеркәсіптің алдында тұрған негізгі міндеттерге бір сатылы беріліс коэффициенттерін арттыру, жүк көтергіштігін арттыру және стандартты беріліс жетектеріне қарағанда тиімділікті арттыру кіреді. Мақалада математикалық және компьютерлік модельдеу нәтижелері келтіріледі, сондай-ақ ЭЦ-ілінісудің эвольвенттік берілісімен салыстырмалы сипаттамасы беріледі. Талдамалы есептеу нәтижесінде редуктордың ЭЦ-ның энергиялық параметрлері анықталды, баламалы кернеулер және беріліс біліктерінің статикалық иілу мәндері алынды. Мақалада ЭЦ ілінісуі бар жаңа беріліс элементтерін статикалық есептеу нәтижелері келтірілген, сондай-ақ ілінісуден туындайтын байланыс кернеулерін компьютерлік модельдеу алгоритмі келтірілген. Компьютерлік модельдеумен айқындалған ЭЦ-дағы түрлі талдамалық әдістемелер мен түйіспелі кернеулер бойынша есептелген дәстүрлі эвольвенттік берудегі түйіспелі кернеулерді салыстырмалы талдау бойынша қорытындыда берудің жаңа түрінің артықшылықтары және оны тау-кен техникасында өткізу тетіктерінде қолдану мүмкіндігі туралы қорытынды жасалады.

**Түйін сөздер:** эвольвенттік-циклоидтық берілістер, эвольвенттік ілінісу, Новиковтың ілінісуі, түйіспелі кернеулер, орталықтан тепкіш күштер.

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### **Математическое и компьютерное моделирование передачи с нетрадиционным зацеплением для привода горной техники**

**Аннотация.** В современной механике передач механизмы передачи играют ключевую роль в преобразовании вращательного движения ведущего вала в вращательное движение другого вала с изменяющимися угловыми скоростями и крутящим моментом. Для достижения оптимальных

конструкций механизмов передачи нового поколения необходимо создавать математические модели их динамического поведения, проводить компьютерное моделирование геометрии зацепления ключевых компонентов и визуализировать процесс работы механизма. Несмотря на широкое использование эвольвентного зацепления в механических передачах, продолжают поиски новых типов зацепления, обладающих преимуществами по сравнению с традиционной системой. Основные задачи, стоящие перед отраслью, включают увеличение передаточного числа в одной ступени, повышение грузоподъемности и эффективности по сравнению со стандартными зубчатыми передачами. В статье приводятся результаты математического и компьютерного моделирования, а также дается сравнительная характеристика ЭЦ-зацепления с эвольвентной передачей. В результате аналитического расчета были определены энергосиловые параметры ЭЦ редуктора, получены эквивалентные напряжения и значения статических прогибов валов передачи. В статье приведены результаты статического расчета элементов новой передачи с ЭЦ зацеплением, а также приведен алгоритм компьютерного моделирования контактных напряжений, возникающих в зацеплении. В заключении по сравнительному анализу контактных напряжений в традиционной эвольвентной передаче, рассчитанных по различным аналитическим методикам и контактных напряжений в ЭЦ зацеплении, определенных компьютерным моделированием делается вывод о преимуществах нового вида передачи и возможности ее применения в передаточных механизмах в горной технике.

**Ключевые слова:** эвольвентно-циклоидные передачи, эвольвентное зацепление, зацепление Новикова, контактные напряжения, центробежные силы.

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